THE ESSENTIALS OF DESCRIPTIVE GEOMETRY

BY

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1908

HILL PUBLISHING COMPANY

505 PEARL STREET, NEW YORK

6 BOUVERIE STREET, LONDON, E. C.

American Machinist — Power — The Engineering and Mining Journal

Hill Publishing Company, New York, U. S. A.

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PREFACE

IT is the general opinion of those who have studied Descriptive Geometry that the subject is a difficult one.

It is spoken of by students as hard to hold on to, slippery, evasive, difficult to apply, easy to forget. And so it is, if the essentials are not thoroughly mastered in the first place.

As a matter of fact the application is superlatively difficult if the essentials are not thoroughly understood; while to the master of these, the application is little short of being a delight.

This book has been written with the single purpose of helping the student to master the essentials.

By means of carefully worded analyses and special illustrations, the essentials are placed before the student in such a way as to make the whole subject easier to comprehend and easier to remember and apply.

G. H. F.

PITTSBURGH, PA., August 25th, 1908.

THE ESSENTIALS OF DESCRIPTIVE GEOMETRY

CHAPTER I

INTRODUCTION

Foreword: Before beginning a study of descriptive geometry, the student should make up his mind to master each phase of it before going on to the next, and to become familiar with the notation and nomenclature by persistently using them.

Much of the nomenclature is new and is of great value in the wording of clear and definite analyses, as well as in the reading of the drawings.

In general, the analysis of a problem is distinctly more important than the example solution shown.

Every analysis has been worded with the idea of making the case clear to the mind *without the help of any drawing.* At the same time very special attention has been paid to the illustrating of such analyses and essentials as are at all difficult of comprehension; and to a very considerable extent the drawings have been so made as to tell their own story *without the aid of the text.* The whole idea has been to make the subject interesting and easy to understand.

The student should solve all the exercises given in the last chapter; he should also assume and solve a variety of cases of each explained problem. There is no other way to master the subject. Solutions drawn freehand are often quite as satisfactory as though drawn mechanically.

A knowledge of "descriptive" must be *acquired;* it cannot be *given;* and it is only by personally hammering away at the essentials that it can be acquired.

Definition: In the name "descriptive geometry" the word descriptive may be taken to mean "having the quality of describing by exact representation."

Descriptive geometry deals with the representation, on a sheet of drawing paper or other flat surface, of geometrical magnitudes, as points, lines, surfaces, and solids, in such a way that their shapes, dimensions, relative positions, and intersections are either fully represented or fully determined.

Reference Planes: As a means of exact representation and determination, we make use of certain planes of projection called "reference planes," whose relation to one another is fixed.

A *plane* is a *flat surface*; it has no thickness and is therefore transparent.

The two principal reference planes are the horizontal (H) and the vertical (V). These, like many other magnitudes used in

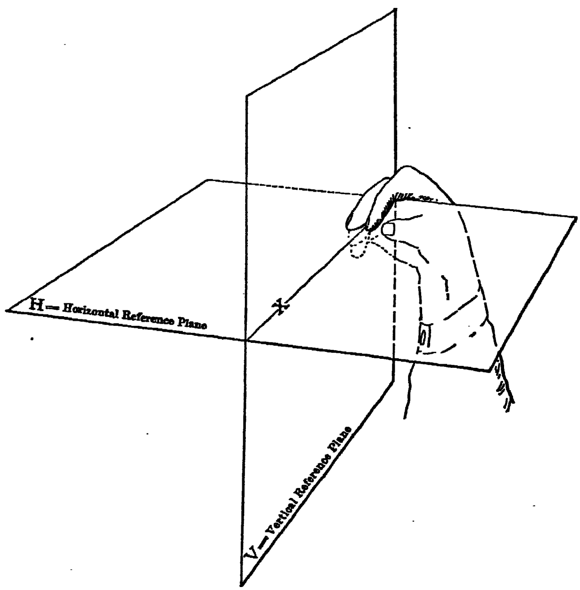


FIG. 1

descriptive geometry, are to be considered as unlimited in extent; that is, extending as far as necessary; having no definite limitations.

In the scheme of perspective illustrating adopted in this book, H and V are generally represented as in Fig. 1—either with or without the hand, the only purpose of which is to give greater realism. The line of intersection of H and V is called X. Drawings of this kind are spoken of throughout as "model drawings."

Representation: In general, magnitudes are represented by their orthographic projections upon, or their intersections with, the reference planes H and V.

The word orthographic means, literally, straight written, from the Greek *ορθός*, straight, and *γραφείν*, write, and indicates that the projections are obtained by straight, parallel projectors, as distinguished from the converging projectors used in perspective projection.

NOMENCLATURE

Much of the nomenclature and many of the abbreviations and symbols used in descriptive geometry are peculiar to the subject; familiarity with them, which is an essential to the comfortable study and understanding of the subject, comes only with their persistent use. It is not necessary to commit them to memory by special effort; in fact such a thing is practically impossible; their persistent use in thinking, writing, and drawing is the best way to fix them in the mind.

ABBREVIATIONS, SYMBOLS, AND NOTATION

The following list is intended for general reference:

H = horizontal reference plane.
V = vertical reference plane.
P = profile reference plane.
X = line of intersection of H and V.
Y = line of intersection of H and P.
Z = line of intersection of V and P.
Hp = H projector; either line or plane; at right angles to H.
Vp = V projector; either line or plane; at right angles to V.
Pp = P projector; either line or plane; at right angles to P.
H.P. = horizontal projection = the trace of an Hp.
V.P. = vertical projection = the trace of a Vp.
P.P. = profile projection = the trace of a Pp.
h.t. = horizontal trace of a line.
v.t. = vertical trace of a line.
p.t. = profile trace of a line.
H.T. = horizontal trace of a plane.
V.T. = vertical trace of a plane.

P.T. = profile trace of a plane.
R = axis of revolution.
θ (Theta) = angle with H.
ϕ (Phi) = angle with V.
β (Beta) = angle with P.
α (Alpha) = angle between two lines.
δ (Delta) = angle between two planes.
Σ (Sigma) = angle between line and plane.
K = true angle between the traces of a plane.
h, v, and p are used as exponents, meaning horizontal, vertical, and profile projections.
$\odot$ = point mark.
$\perp$ = at right angles or perpendicular to.
$\parallel$ $\neq$ parallel.
Sub-zero means "revolved position"; as a_o.

Points in space are lettered with lower case letters, a, b, c, d, etc.; their projections are marked with the proper exponent, as a^h, a^v, a^p; a^h means horizontal projection of point *a*.

Lines in space, of indefinite length, are lettered with capital letters, as A, B, C, etc.; their projections A^h, A^v.

A line determined by two points a and b is called the line ab; a^hb^h, a^vb^v.

A point determined by the intersection of two lines A and B is point $\overline{AB}$; $\overline{AB}^h$, $\overline{AB}^v$.

Planes in space are numbered 1, 2, 3, etc. Their traces are 1^h, 1^v, etc.

The line of intersection of two planes 2 and 4 is line $\overline{24}$; $\overline{24}^h$, $\overline{24}^v$.

The point of intersection of three planes 1, 2, 3 is point $\overline{123}$; $\overline{123}^h$, $\overline{123}^v$.

The plane of three points a, b, c is the plane $\overline{abc}$.

The plane of two lines A and B is the plane $\overline{AB}$.

The plane of line A and point c is plane $\overline{Ac}$.

The point of plane 2 and line B is point $\overline{2B}$.

CHOICE OF NOTATION LETTERS

To avoid confusion and ambiguity, notation letters should always be taken from the following table; only those letters are included that are suitable for notation purposes.

Reserved letters.	Reference plane notation.	Reference plane axes.	Points in space.	Lines in space.	Angles.	For miscellaneous use.
A			a	A	θ	
B			b	B		
C			c	C	ϕ	J; j
D			d	D		
E			e	E	β	L
F			f	F		
G			g	G	α	Q
H	H. h				δ	
K						
M			m	M		r
N			n	N	Σ	
P	P. p				κ	S, s
V	V. v					
X		X				
Y		Y				
Z		Z				

SPECIAL LINES

In descriptive geometry the exercise of a little ingenuity in the use of lines of varied weight and character will often make all the difference between an obscure and a clear drawing. Fig. 2 shows three special lines and names them; it is convenient to

Dotted line:	For projection lines only.
Dot and dash line:	For auxiliary or solving plane traces.
Dash line:	For auxiliary or solving lines.

FIG. 2

allow considerable latitude in their use, as will be seen later. For other purposes the full line is used, varied in weight to suit conditions, except where one of the regular lines of the alphabet of mechanical drawing is preferable.

In descriptive drawings, where it is seldom necessary to consider a line as hidden or any magnitude as opaque, and in which different parts of the same magnitude are not always in the same dihedral angle, the reading of a drawing is finally dependent upon the notation rather than upon the character of the lines.

ELEMENTARY DEFINITIONS AND NOMENCLATURE

Projections: If through a point in space a straight line be drawn perpendicular to a given reference plane, the point where the line pierces the plane is called a projection of the point.

In Space: The term "point in space" signifies that the position of the point is a matter of choice, entirely independent of any limiting conditions. The terms "line in space," "plane in space," have the same significance.

Referring to Fig. 3: The perpendicular through the point in space is called a "projector"; if the reference plane is horizontal (H) the projection obtained is called a "horizontal projection" (H.P.), and the projector is called an "H projector" (Hp). Hp is *never* to be read "horizontal" projector, but *always* "H projector"; it takes its name from the reference plane that it is perpendicular

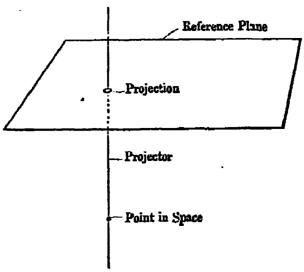


FIG. 3

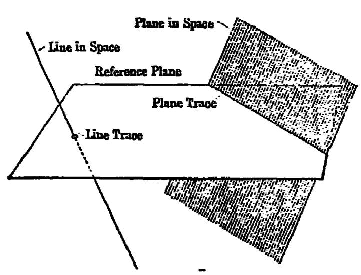


FIG. 4

to; so that an H projector line (Hp) is vertical, and a V projector line (Vp) is horizontal.

The Trace of a Line: The point where a line in space intersects a reference plane is called the *trace* of the line in that plane; or more concisely, line trace (see Fig. 4).

Trace: One of the meanings of the word trace, as given in the Century Dictionary, is "the mark which indicates the course pursued by any moving thing." We may look upon the trace of a line then as the mark it leaves in passing through a reference plane.

The Trace of a Plane: The line where a plane in space intersects a reference plane is called the "trace of the plane," or "plane trace" (see Fig. 4).

To distinguish between traces in H and in V they are called horizontal and vertical traces respectively, H.T., V.T.

CHAPTER II

THE THIRD ANGLE

The Dihedral Angles: By their intersection, H and V form four right-angular spaces, called the first, second, third, and fourth dihedral angles (see Fig. 5).

Dihedral means having two sides.

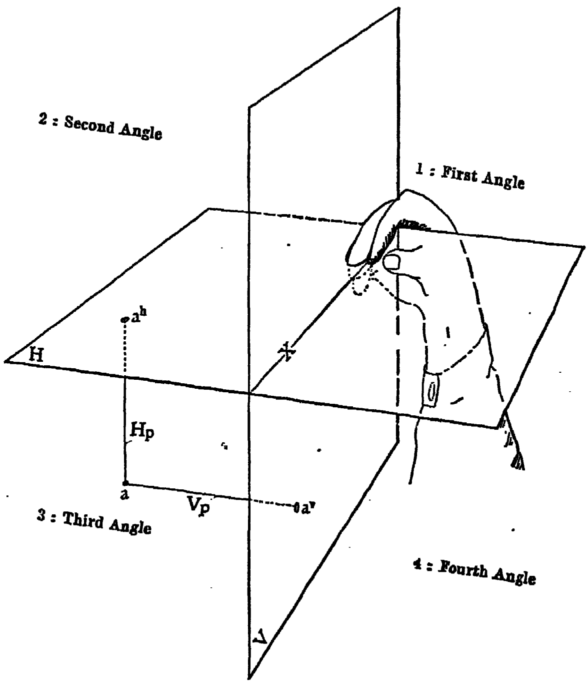


FIG. 5

Third-Angle Projection: When a magnitude to be represented is placed wholly in the third dihedral angle, its projections are called "third-angle."

In mechanical drawing a large majority of the drafting rooms of this country have adopted third-angle projection.

In descriptive geometry *all four angles have to be used,* because lines and surfaces must often be considered as unlimited in extent

and as *passing through the reference planes*; but in the majority of cases it is possible to confine a solution either entirely or largely to *one* of the angles. In this book the third angle is used, because the projections are then similar in their grouping to those obtained in regular mechanical drawings. The advantages of using the same angle for both descriptive and mechanical drawings are too obvious to need more than passing mention here. By using third angle for one and first for the other, the mind is quite unnecessarily confused.

REVOLUTION OF "V"

Referring to Fig. 5, a is a point in the third angle; a^h is its horizontal projection (H.P.); a^v is its vertical projection (V.P.).

In order that these projections may be shown on a single flat surface, as a sheet of drawing paper mounted on a drawing board, the vertical reference plane V is revolved about X so as to close the second and fourth angles to zero, and open the first and third to 180°.

Fig. 6 is a model drawing so contrived as to show the evolution of the representation, by descriptive geometry, of a point a in the third angle, from the point itself to its projections on the board.

Nomenclature: The full meaning of "the board" throughout this book is "the flat surface of the sheet of paper mounted on the drawing board."

If carefully studied *now*, this model drawing will fix certain habits of mind that will help:

1. H is to be considered as coincident with the board, whether this be held in the horizontal, vertical, or sloping position.

2. V is perpendicular to the board, and its position is indicated by its line of intersection with H, which line is called X.

3. The T-square blade, as it lies on the board in this model drawing, is *on* H and *in* the first angle.

4. As the draftsman stands at the board, the part of him that is above the board is in the first angle; the part of him below the board is in the fourth angle.

5. Above the board and behind V is the second angle.

6. The third angle, with which the student will be chiefly interested, is below the board and behind V.

7. The arrows W show which way V is revolved about X until it coincides with H, or "the board." The part of V that is

below the board swings up toward the draftsman; the part above the board swings down away from the draftsman.

Note: Remember that a plane has no thickness, and that after revolving, V does not lie on top of H or under it, but coincides with it.

8. In a descriptive drawing the whole surface of the board is both H and V. It is absolutely essential to get this idea clearly in mind.

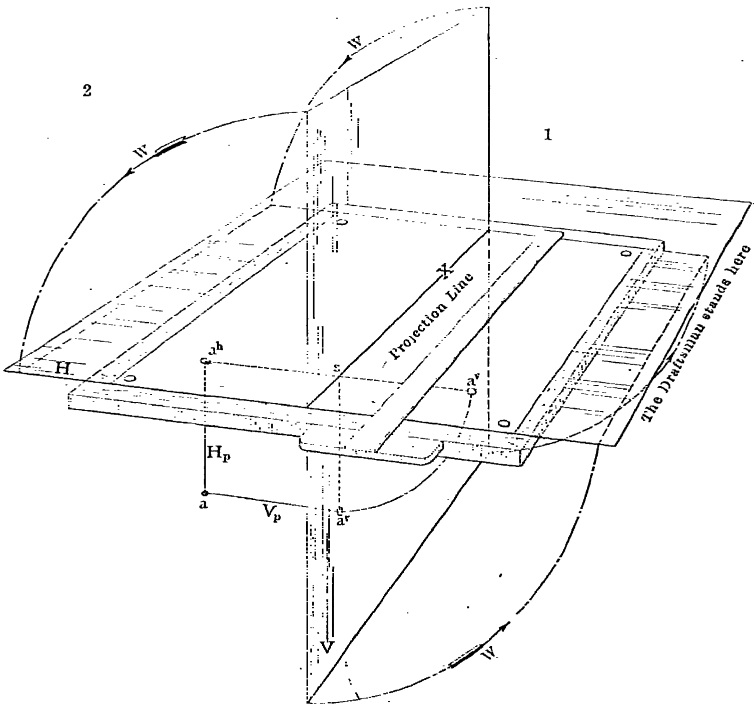


FIG. 6

9. a^h s a^v is called a "projection line," which can be defined as the projection of both projectors; or we may say that a^hs is the horizontal trace (H.T.) of the plane of both projectors, and that a^vs is the vertical trace (V.T.) of the plane of both projectors. The projection line is of course at right angles to X.

10. The distance of the point a from V. = aa^v = a^hs.

The distance of the point a from H. = aa^h = a^vs.

11. The horizontal projection a^h can be legitimately defined as the trace of the H projector line. Similarly a^v = trace of Vp.

12. When a magnitude is in the third angle its H.P. (horizontal projection) is above X on the drawing, and its V.P. (vertical projection) is below X on the drawing.

13. Fig. 7 means the same as Fig. 6.

14. As you look at a^h, in Fig. 7, get the idea that you are looking, *through the drawing board*, at the point a itself, which is below the board at a distance sa^v from the surface of the drawing paper.

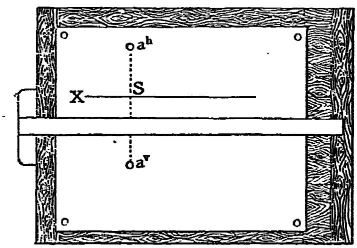


FIG. 7

15. The draftsman, as he stands at the board, should think of the drawing paper as H *in fact*; it is also V after V has been revolved into the board about X.

16. In all our model drawings we shall look at the dihedral angles from the T-square end of the drawing board; from this position V revolves counter-clockwise about X into H. Looking from the other end, V revolves clockwise.

CHAPTER III

REPRESENTATION OF POINTS IN SPACE

Analysis: The projection of a point is the trace of its projector line. Specifically, the H.P. of a point is the trace of its Hp line; the V.P. of a point is the trace of its Vp line.

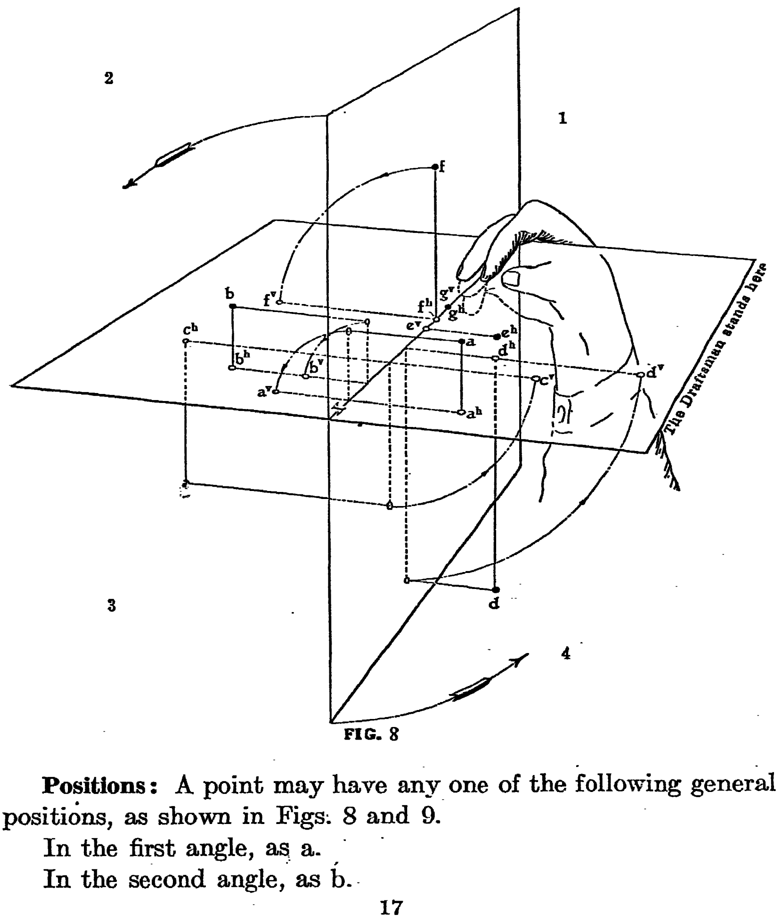


FIG. 8

Positions: A point may have any one of the following general positions, as shown in Figs. 8 and 9.

In the first angle, as a.

In the second angle, as b.

In the third angle, as c.
In the fourth angle, as d.
In H, as e.
In V, as f.
In H and V = in X, as g.

To represent a point, two projections are always required, and two only. When a point is in X, its H and V projections

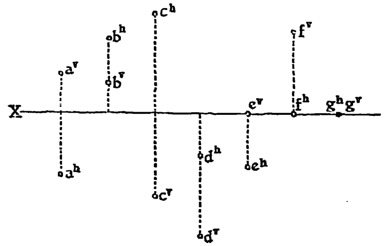


FIG. 9

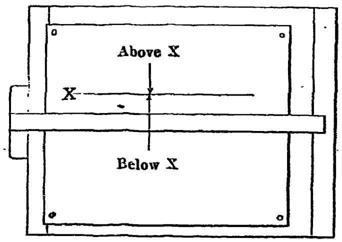


FIG. 10

coincide and must be lettered for both, as point g in Figs. 8 and 9; otherwise the point is not represented.

Notation: In descriptive geometry, projections have no independent meaning unless they carry complete notation.

By Inspection of Fig. 9 we observe the following:

A point in first angle: H.P. is below X; V.P. is above X.

A point in second angle: Both projections are above X.

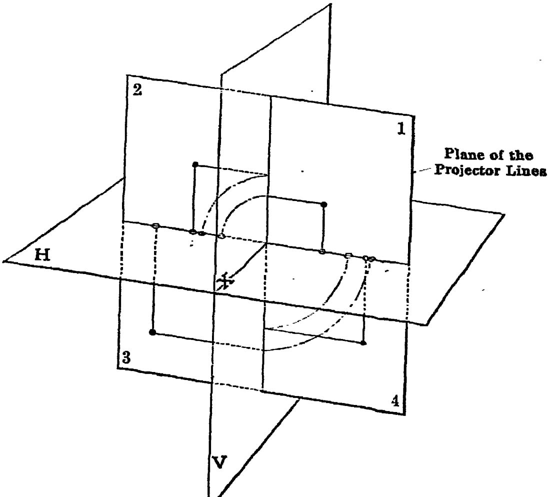


FIG. 11

A point in third angle: H.P. is above X; V.P. is below X.

A point in fourth angle: Both projections are below X.

A point in H: V.P. is in X.

A point in V: H.P. is in X.

A point in both H and V: The point itself is in X, and coincides with its two projections.

Above and Below X: The terms "above X," "below X," refer to positions on the board (see Fig. 10). The line X is generally drawn horizontal, so that, as the draftsman stands at the board, he sees that some projections come above X, some below, and some in X. This is the only sense in which these terms are used.

A point in the third angle is underneath that portion of the board that is above X.

The Plane of Projector Lines: Get the idea of a point in space as lying in the plane of its projector lines, as in Fig. 11. Now move the point about this plane and realize that in all positions its projections lie in the same projection line, and that every position is fully represented if the two projections are marked with correct notation.

CHAPTER IV

REPRESENTATION OF LINES IN SPACE

Nomenclature: Unless a line is described as curved it is always understood to be straight.

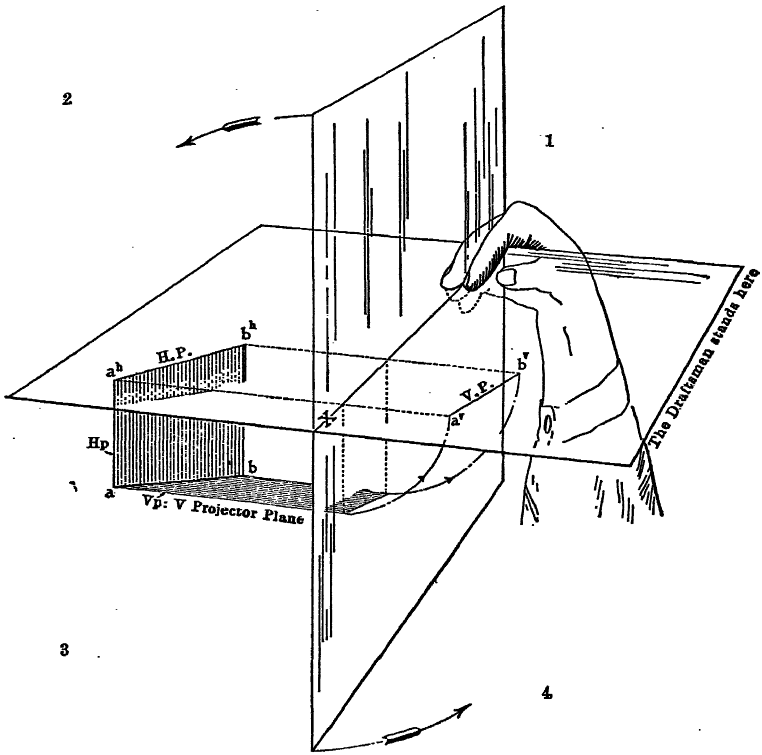


FIG. 12

Notation: Lines in space are designated by capital letters, as A, B, C; their projections by adding the proper exponent letter, as A^h, A^v.

A line may be determined by two points in it, as *a* and *b*; it is then called the line ab; a^hb^h, a^vb^v.

Length: In descriptive geometry, lines are generally understood to be of unlimited length, extending indefinitely both ways.

Analysis: The projection of a line is the like trace of its projector plane. Fig. 12 is an illustration of this analysis. Fig. 13 has the same meaning.

Projections: To obtain the projections of a line: Take two points in it and find their projections; the straight line through these point projections is the projection of the line.

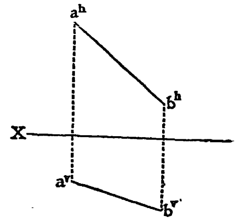


FIG. 13

Get the idea that in order to handle a line we must have the projections of two points in it.

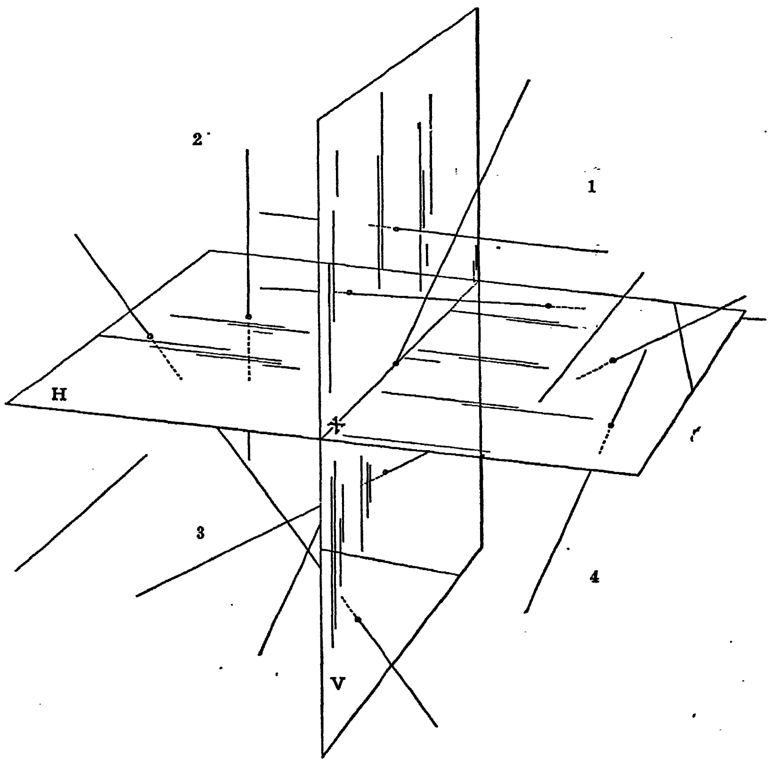


FIG. 14

Positions: With relation to H, V, and the four dihedral angles, a line in space may be placed in a great variety of ways, as illustrated in Fig. 14.

CHAPTER V

LINE TRACES

Definition: The point where a line in space pierces a reference plane is called the trace of the line, or, more concisely, the "line trace."

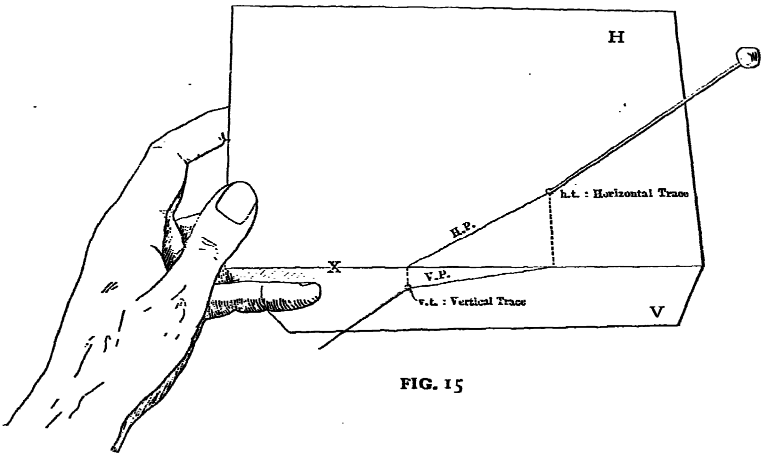


FIG. 15

Experiment I: Take a sheet of paper; crease it along a horizontal line and fold the lower half down in imitation of V. Now, to represent a straight line, take your scarf pin (or a hat pin will be better) and pass it down through H so as to come out through V as shown in Fig. 15.

The part of the pin that you cannot see is in the third angle; the point where it pierces, or intersects, H is the horizontal trace (h.t.); the point where it pierces V is the vertical trace (v.t.).

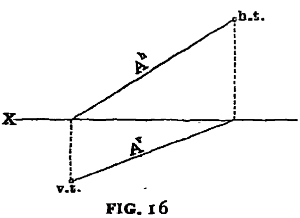


FIG. 16

The part between the two traces has projections H.P. and V.P.

Fig. 16 means the same as Fig. 15, and is a "descriptive drawing."

Analysis: By inspection of Figs. 15 and 16: The h.t. is at the intersection of the H.P. and the projection line from where the V.P. meets X.

The v.t. is at the intersection of the V.P. and the projection line from where the H.P. meets X.

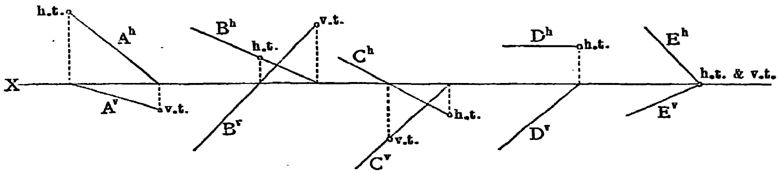


FIG. 17

Note: The h.t. (horizontal trace) is always in the H.P. (horizontal projection). The v. t. (vertical trace) is always in the V.P. (vertical projection).

In Fig. 17 several varieties of line-trace solutions are shown.

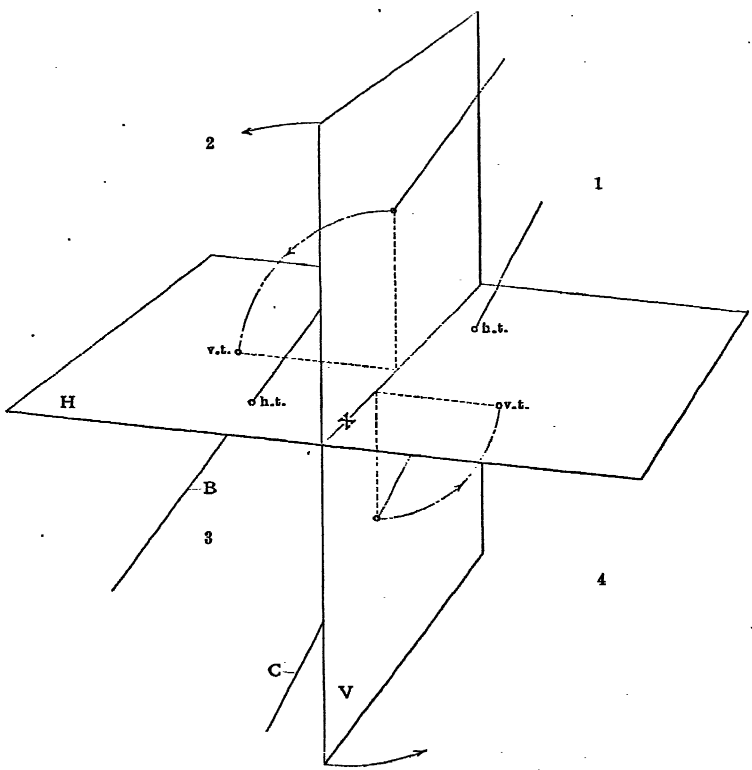


FIG. 18

They are all true to the same analysis; D is parallel to V, and therefore has no v.t.; E passes through X, and therefore its two traces coincide in X.

Fig. 18 shows how it happens that both traces of line C, Fig. 17, come below X on the board; it is because the line passes through, or crosses, the fourth angle.

When the line crosses the second angle, both traces come above X (see line B, Figs. 17 and 18).

Of course, if a line is parallel to both planes, it has no traces.

A line is determined if its two traces are given.

It will pay the student to be very thorough in his study of line traces; he should try several cases, drawing the solutions freehand.

Descriptive geometry is difficult chiefly, perhaps, because of the great *apparent* variety of solutions of the same problem. Fig. 17 illustrates this statement.

The student should continually fall back upon the general analysis of a problem, and it is an excellent habit to have in mind a simple example solution to which to refer all cases. Of the cases solved in Fig. 17, the first one—line A—is a good example solution. By inspection we see that all the solutions are true to the same analysis.

CHAPTER VI

REPRESENTATION OF PLANES IN SPACE

Definition: The line where a plane in space intersects a reference plane is called a trace of the plane, or, more concisely, a "plane

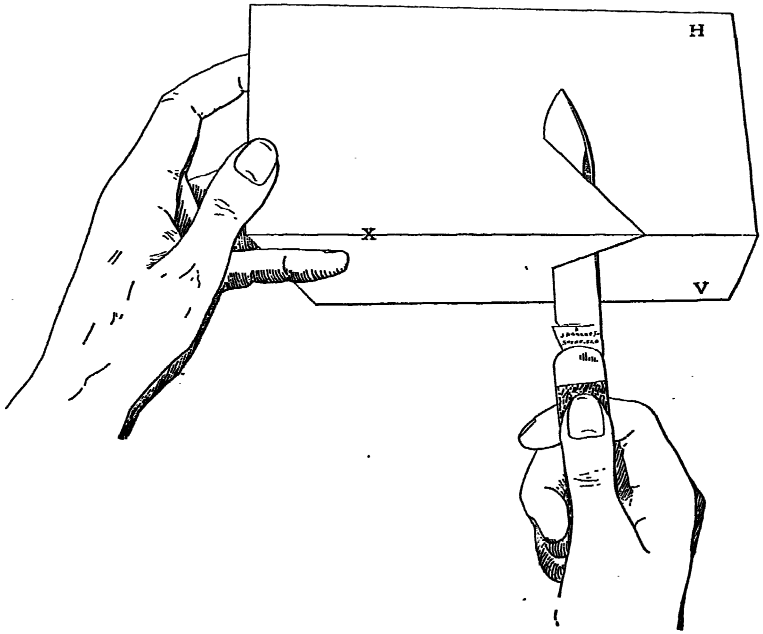


FIG. 19

trace." A plane in space is represented by its trace or traces, and is called a "trace plane."

Notation: Trace planes are designated by number, as 1, 2, 3; their traces are 1^h, 1^v, etc.

Experiment II: Take a sheet of paper, and fold it, as before, to form the third angle. Now, with a sharp knife, make a slanting cut as in Fig. 19, and into this cut slide a thin card, as in Fig.

20; the card represents a plane in space; the part of it that you cannot see is in the third angle, and the H and V traces of this portion are marked H.T. and V.T. respectively. **By Inspection,** the traces intersect in X.

Intersection of Traces: If the traces of a plane are not parallel, they intersect in X.

Proof: By geometry, H and V intersect in the straight line X. A plane that is not parallel to a given straight line intersects that line in one point only. If the two traces do *not* intersect

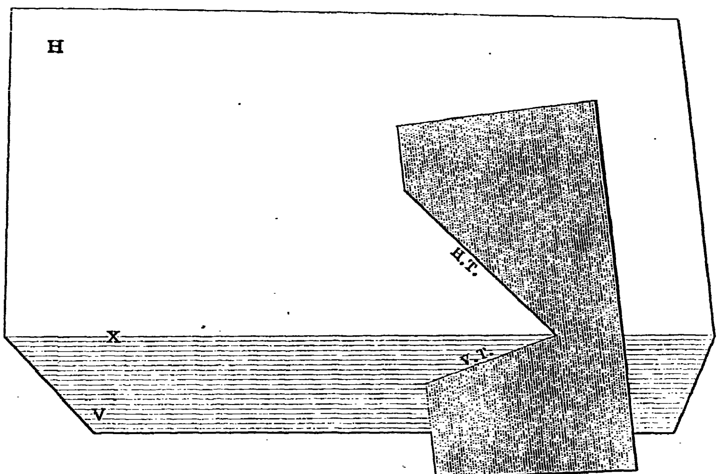
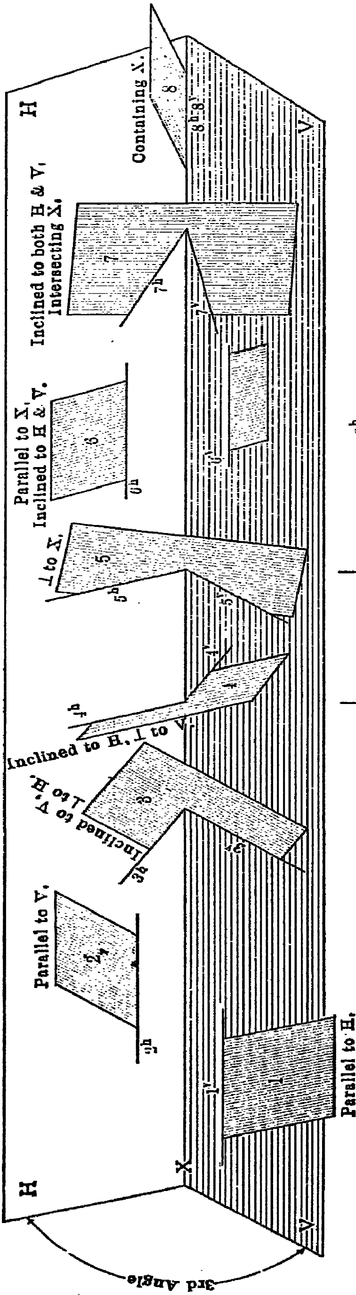


FIG. 20

in X, then they will intersect X in two points, and we shall have a plane intersecting a straight line in two points, which is impossible.

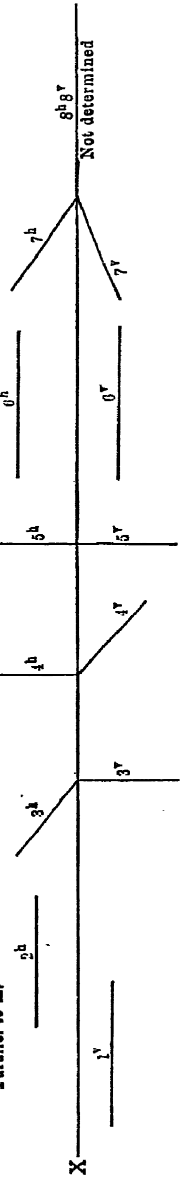
Positions: With relation to the reference planes, a plane in space may occupy an infinite number of positions, but every position will belong to one or other of the following general cases as illustrated in model drawing Fig. 21, and descriptive drawing Fig. 22, which have exactly the same meaning:

Plane 1. Parallel to H.
Plane 2. Parallel to V.
Plane 3. Perpendicular to H and inclined to V.
Plane 4. Perpendicular to V and inclined to H.
Plane 5. Perpendicular to both H and V, and therefore to X.
Plane 6. Parallel to X and inclined to H and V.
Plane 7. Inclined to both H and V; intersecting X.
Plane 8. Containing X and inclined to both H and V.



H
H
V
V
X
Containing X.
Inclined to both H & V,
Intersecting X.
Parallel to X,
Inclined to H & V.
⊥ to X.
Inclined to H, ⊥ to V.
Inclined to V, ⊥ to H.
Parallel to V.
Parallel to H.
3rd Angle

FIG. 21



X
Not determined

FIG. 22

Trace Projections: Each trace of a plane is a line of the plane. The vertical projection of the H.T. is in X. The horizontal projection of the V.T. is in X. It is very necessary to appreciate this fact. It is illustrated in Fig. 23, in which the line A is parallel to the line 2^h of the trace plane 2.

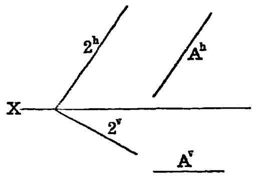


FIG. 23

Referring now to model drawing Fig. 24, 5 is a trace plane, similar to 7 in Figs. 21 and 22, inclined to H, V, and X. Its traces intersect in X and are correctly marked. The revolving of V brings 5^v into the board—coincident with H, as shown.

This plane is seen to pass *through* H and V into all four dihedral

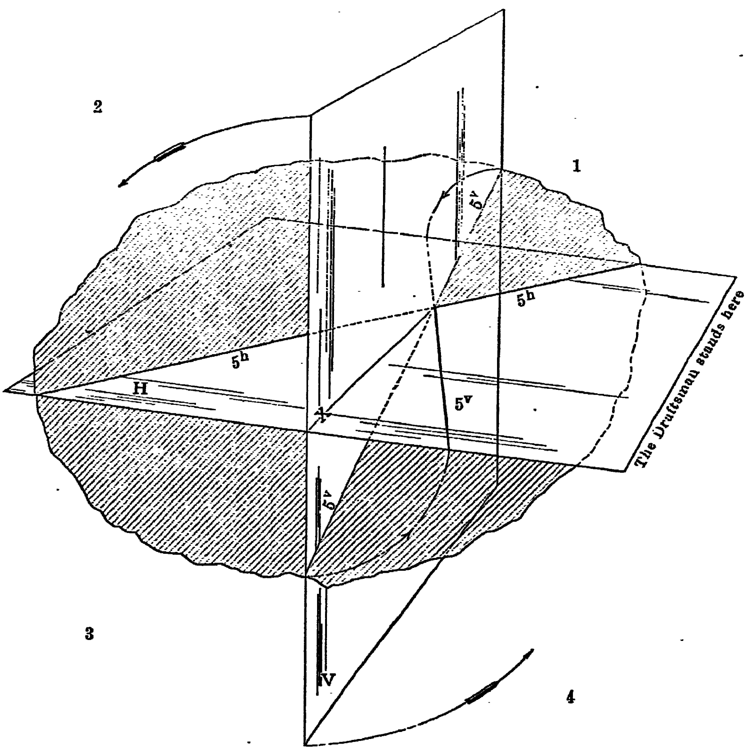


FIG. 24

angles; and is thus divided by its traces into four portions—one portion in each angle.

Fig. 25 is a descriptive drawing of this same plane, representing all four portions of it.

In Fig. 26 the portions are shown separately; these four drawings have one and the same meaning, and are all duplicates of Fig. 25.

The Term Oblique: It is sometimes convenient to give special significance to a word; in this book the word oblique, when used to describe a trace plane, means inclined to both H and V and intersecting X. Plane 5 in Figs. 24 to 26 is thus an oblique plane. When referring to a straight line, the same term means inclined to H, V, and X.

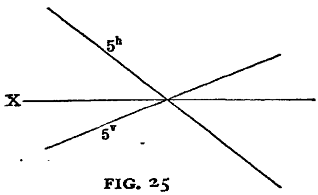


FIG. 25

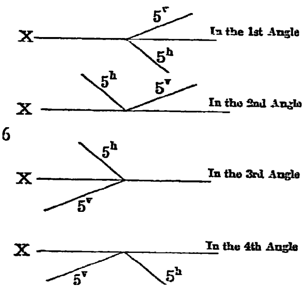


FIG. 26

CHAPTER VII

QUESTION-AND-ANSWER REVIEW

1. In order to represent the shapes, dimensions, and relative positions of magnitudes we make use of what?

Ans. Reference planes whose relation to one another is fixed.

2. What is a plane?

Ans. A perfectly flat surface.

3. Is it transparent and why?

Ans. Yes; because it has no thickness.

4. How are magnitudes represented?

Ans. By their orthographic projections upon or their intersections with the reference planes.

5. In order that projections on the two reference planes may be represented on one surface—as the board—what do we do?

Ans. We revolve the vertical reference plane into the board.

6. How does V revolve with relation to H?

Ans. It revolves about the line of intersection X until the second and fourth angles become zero.

7. What do we mean by third-angle projection?

Ans. We mean that the magnitude is in the third dihedral angle, and that in the "drawing" of it the H projection comes above X and the V projection below X.

8. How are points in space designated?

Ans. By lower-case letters, a, b, c, etc. Their projections by the same letters—with ^{h} or ^{v} as an exponent—as a^h, a^v, etc.

9. How are lines of indefinite length designated?

Ans. By capital letters, as A, B, C, etc. Their projections are lettered A^h, A^v, etc.

10. A line determined by two points a and b is designated how?

Ans. As the line ab; a^hb^h, a^vb^v, etc.

11. A point determined by the intersection of two lines, as A and B, is called the point what?

Ans. It is called the point $\overline{AB}$; $\overline{AB^h}$, $\overline{AB^v}$.

12. How are planes in space represented, and what are they called?

Ans. By their traces; they are called trace planes.

13. How are trace planes designated?

Ans. By the numerals, as 1, 2, 3, 4.

14. Define the horizontal projection of a point.

Ans. It is the trace of its Hp line.

15. What is the vertical projection of a line?

Ans. It is the vertical trace of its Vp plane.

16. If a plane is inclined to both H and V and is not parallel to X, where do its traces *always* intersect?

Ans. In X.

17. With relation to the board, describe the position of a point in the third angle.

Ans. Underneath that portion of the board that is above X.

18. When a line of definite length is wholly in the second angle, where do you know that its projections will lie?

Ans. Both above X.

19. When a plane is parallel to H and below it, how is it represented?

Ans. By its V.T., which is below X and parallel to it.

20. When is a line of indefinite length not determined by its H and V projections?

Ans. When it lies in a plane at right angles to X.

21. When is a trace plane not determined by its H and V traces?

Ans. When the plane contains X.

22. Which of the dihedral angles does the draftsman at the board occupy?

Ans. The first and fourth.

CHAPTER VIII

THE PROFILE PLANE

When a line of indefinite length lies in a plane at right angles to X, its H and V projections are in the same straight line at right angles to X and fail to determine it. Similarly, when a plane contains the line X, its H and V traces coincide with X and fail to determine it.

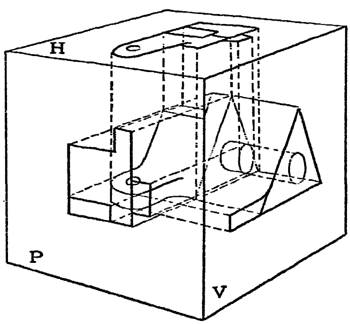


FIG. 27

In general, whenever the H and V projections or traces fail to determine a magnitude, it is necessary to make use of an "auxiliary" reference plane.

Although such a plane may bear any relation whatever to H or V, the one most commonly convenient is perpendicular to both H and V, and therefore to X; and this particular auxiliary reference plane is called the profile, or P.

In mechanical drawing we know that two views of an object often fail to represent it fully; a third view is then drawn; the three views, which we may call top, front, and end, are exactly

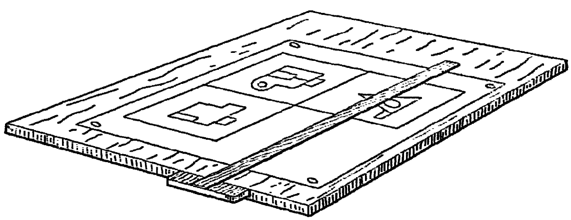


FIG. 28

equivalent to the H, V, and P projections in descriptive geometry. In Fig. 27 three such projections are shown, and Fig. 28

shows these projections on the board, as they might be arranged in a regular mechanical drawing.

In descriptive geometry, when the profile plane is used, it must

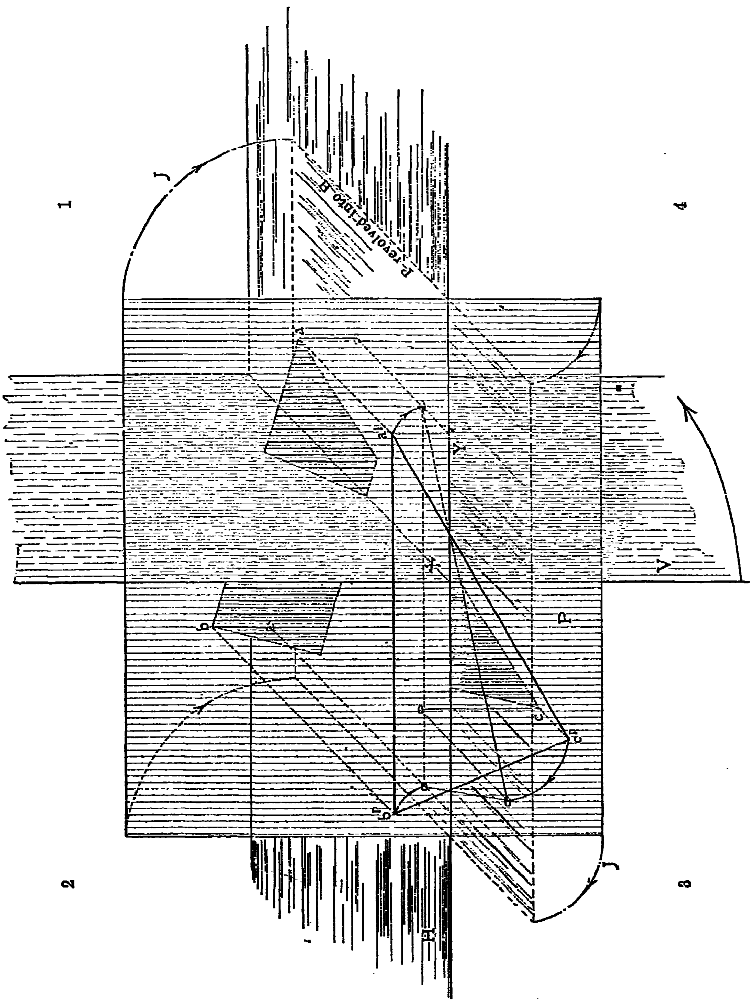


FIG. 29

be remembered that *the whole surface of the board is three planes*, H, V, and P.

Notation: The line of intersection of H and P is called Y. The line of intersection of V and P is called Z.

Revolution of P: The profile reference plane P can be re-

volved into H in two ways; either directly about axis Y, or first into V about axis Z and then *with* V into H about axis X. In this

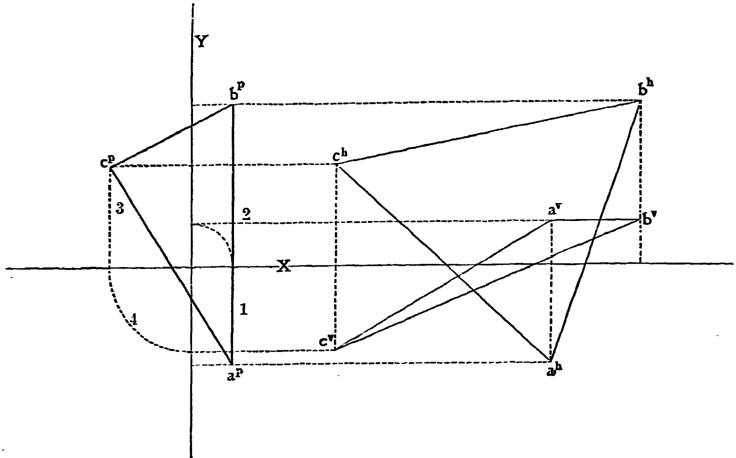


FIG. 30

book the direct revolution about Y is generally used, because it is a simpler process.

The profile plane is revolved so as to open the third and fourth angles and close the first and second; the *direction* of revolution

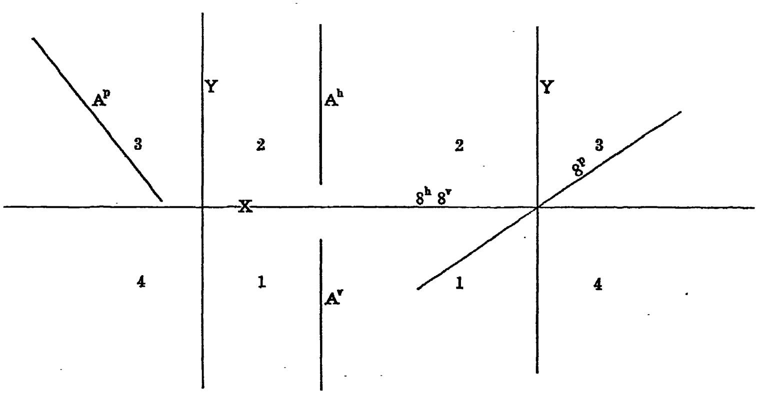


FIG. 31

depends upon the position of the magnitude to be projected; if the magnitude is to the right of P, P is revolved clockwise; if to the left, counter-clockwise.

Adhering to this method of revolving both V and P directly

into H about axes X and Y respectively, the P projection of a point will always be on the same side of X as the H projection, and its distance from Y will be the same as the distance of the V.P. from X.

In general, for all projections, the board *is* H, and V and P are revolved into it about axes X and Y.

Fig. 29 shows by means of a model drawing the general relations of a magnitude in space to the three reference planes H, V, and P. The triangle abc has vertex a in the first angle; b in the second; c in the third. The P projection of this triangle is lettered a^p, b^p, c^p. The direction of revolution of P about Y into

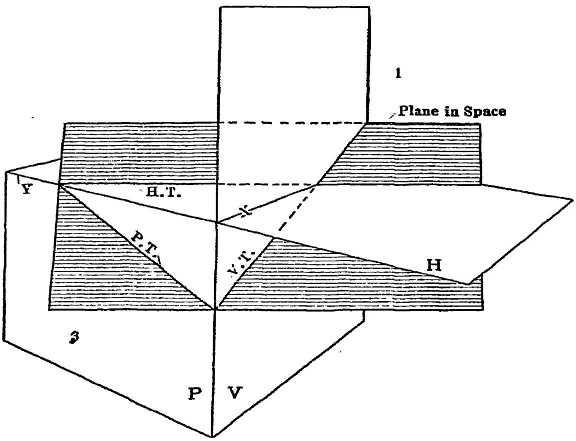


FIG. 32

H, or the board, is indicated by arrows J, and the revolved position of the triangle in P is clearly marked, an arrow from each vertex showing its revolution about Y as an axis.

Fig. 30 is a regular descriptive drawing of the same triangle in the same position in space.

Fig. 31 shows a line A and a plane 8 represented with the aid of profile planes; note that references to H and V alone fail to represent these magnitudes.

Fig. 32 is a model drawing showing the relations of a trace plane to the three reference planes H, V, and P.

CHAPTER IX

PROJECTED VIEWS

THERE are two ways in which any required view of an object can be obtained; or, to put it in other words, in which an observer may see any required view of an object.

First. We may consider the object itself as stationary and the observer as moving about so as to see the different views he requires.

Second. We may consider the observer as stationary and the

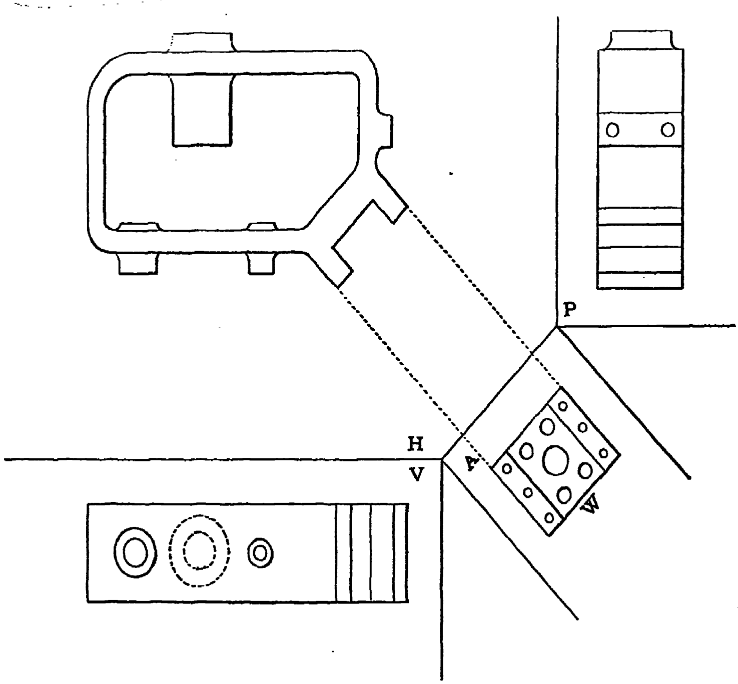


FIG. 33

object as moved into different positions so as to exhibit different views of itself.

In mechanical drawing, the first of these methods is generally used, which means that for every different view there is a different reference plane.

Fig. 33 shows four views of an object, as they might be arranged in a mechanical drawing. The view marked W was obtained

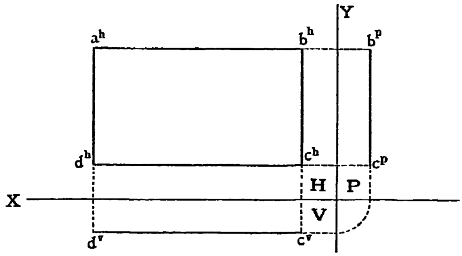


FIG. 34

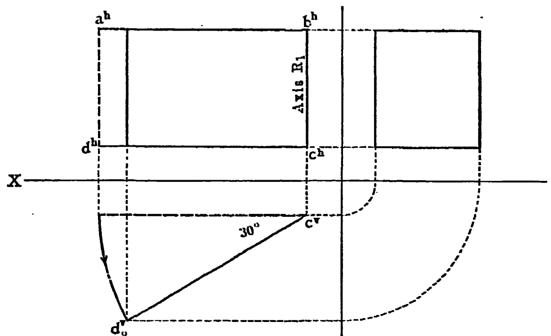


FIG. 35

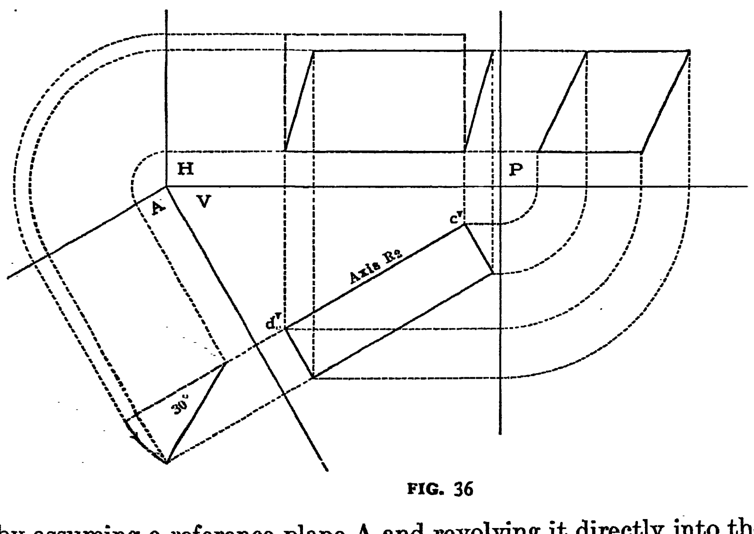


FIG. 36

by assuming a reference plane A and revolving it directly into the board. Any required view can be obtained by this process; it

is only necessary to assume a reference plane at right angles to the direction in which the view is to be projected.

In descriptive geometry, the second method is generally used. The body is revolved about suitable axis or axes until its position with relation to the fixed reference planes is such as to give the required projections upon them.

The process of changing position by revolving about axes is illustrated in Figs. 34 to 37. In Fig. 34 three views are shown

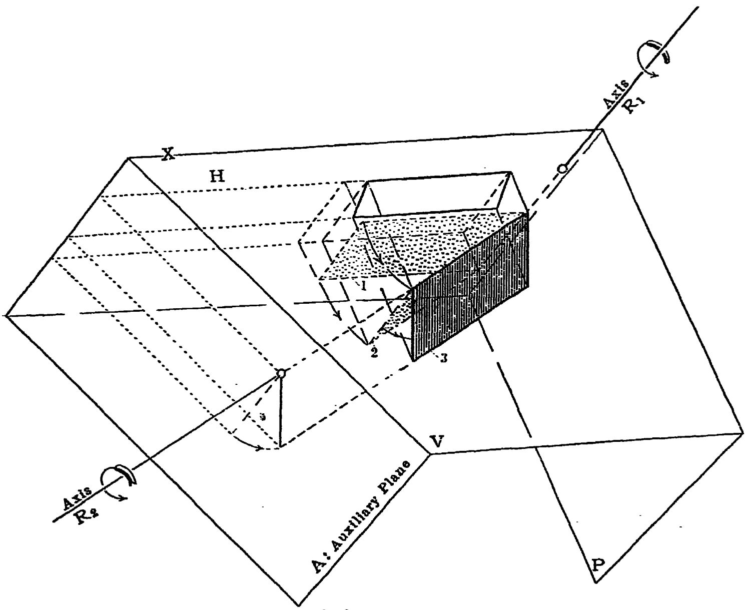


FIG. 37

of a flat rectangular card, thickness neglected. In Fig. 35 this card is shown revolved through 30° about the edge bc as an axis. This is an example of simple change of position by revolving about an axis at right angles to V. Note that all three views of the card are changed.

If now it is required to make a second change of position by revolving the card about axis cd_o, it will evidently be necessary to make use of an auxiliary reference plane A, Fig. 36, preferably at right angles to the axis because, as seen through this plane, the motion of the card will be circular and therefore easy to draw; if any other plane were taken, not at right angles to cd_o, the projected motion would be elliptical.

Fig. 36 shows the second change of position; all three views are again changed.

Fig. 37 is a model drawing showing the double change of position as effected in Figs. 35 and 36, from 1 and 2 about axis R_1, and from 2 to 3 about axis R_2. This model drawing is intended to exhibit the principle of change of position about axes of revolution.

CHAPTER X

THE EXHIBITING OF TRUE DIMENSIONS BY THE USE OF AXES OF REVOLUTION

In descriptive geometry axes of revolution are frequently used, especially when true dimensions are to be exhibited for direct measurement.

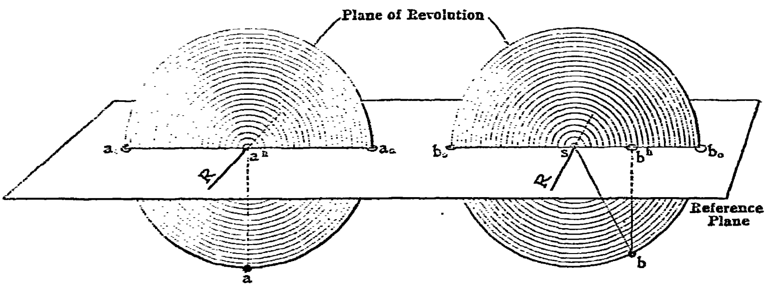


FIG. 38 FIG. 40

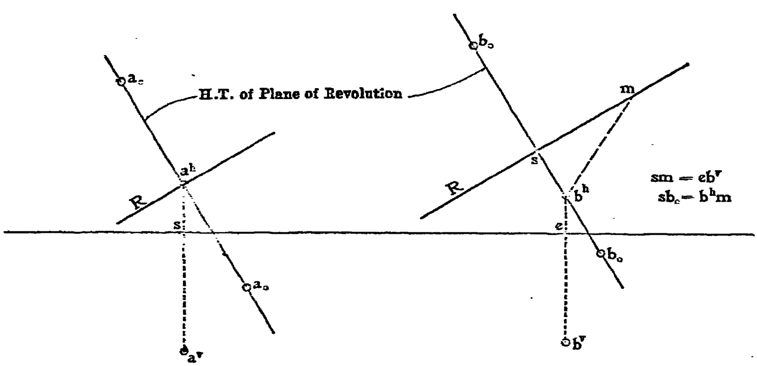


FIG. 39 FIG. 41

The principal fact to bear in mind, in all problems where axes of revolution are used, is that every point in a revolving magnitude moves in a plane at right angles to its axis.

Nomenclature: The revolved position of a magnitude is indicated by adding sub-zero to its notation, as a_o, B_o.

Problem 1: To revolve a point into a reference plane about an axis R in the reference plane.

Analysis: The revolved position of the point lies in the like trace of the plane of revolution at a distance from R equal to the radius of revolution.

Solutions: Referring to Figs. 38 and 39, the point a is vertically under the axis R; a^h is therefore in R; the radius of revolution equals aa^h in Fig. 38, or sa^v in Fig. 39; and the two revolved

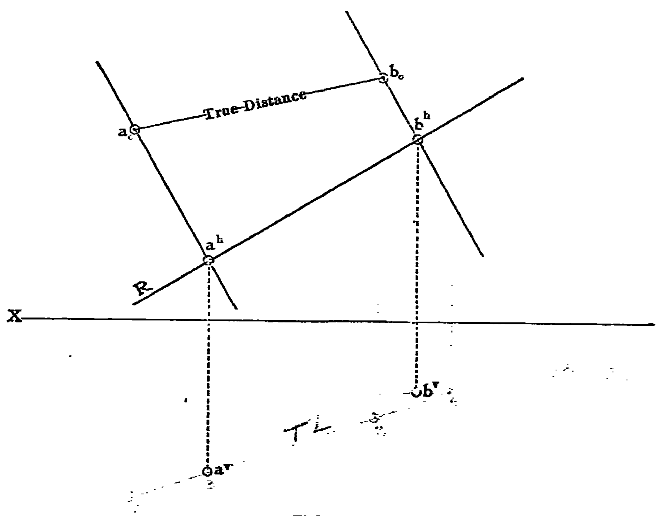


FIG. 42

positions of a, which are marked a_o are a distance from R equal to the radius of revolution.

In Figs. 40 and 41 the point b is not vertically under the axis of revolution; the horizontal projection b^h is therefore not in R; the radius of revolution sb, Fig. 40, is the hypotenuse of a right-angle triangle whose sides are sb^h (the distance of b^h from R) and bb^h (the distance of b from the reference plane containing R).

True Dimensions: The principal use of axes of revolution is to exhibit for direct measurement the true dimensions of magnitudes that are represented by their projections on the reference planes. A simple example will suffice to illustrate; many applications will be found later.

Problem 2: In Fig. 42 the points a and b are represented by their H and V projections; to find the distance between them.

Analysis: Revolve both points into one of the reference planes about an axis R containing both projections, and the true distance between the points will be exhibited for direct measurement.

Solution: The solution for this rather cumbersome analysis is shown in Fig. 42. The axis R contains a^h and b^h; each point is revolved into H, and the true distance between a and b is exhibited for direct measurement at a_o, b_o.

CHAPTER XI

SPECIAL NOTATION SIGNS AND THE READING OF DESCRIPTIVE DRAWINGS

So far we have been dealing chiefly with " means of representation " as used in descriptive geometry. We are now prepared to discuss the analyses and solutions of all the elementary problems concerning the relations of points, lines, and surfaces.

The solution of a problem can be explained in two ways: First, in the abstract, by means of an analysis, in which no special conditions are assumed. Second, in direct connection with assumed conditions and a drawing.

Problems: The word problem means a question proposed for discussion and solution; a problem may be stated in the abstract or in connection with a drawing for direct solution. In a problem, certain conditions are assumed or data given, from which some other datum or data must be obtained by analysis and solution.

Analyses: Every solution depends primarily upon an analysis, which may be defined as a general statement in explanation of all solutions of the same problem.

Solutions: A solution may be defined as an illustration of an analysis. There is often a great variety of solutions, all true to the same analysis.

Notation Signs: When a problem is stated for direct solution, or where an example solution is drawn, it is always stated what data are given and what are to be solved for, or obtained by solution.

There are thus two kinds of data, "given," and "obtained." To distinguish between these, the solution drawings in this book carry certain special notation signs, which are of great importance because they make the drawings easier to read.

All given-data notation is underlined, as $\underline{A^h}$, $\underline{3^v}$, $\underline{m^h}$, etc.; all obtained-data notation is marked with a zero prefix, as $^{o}A^h$, $^{o}3^v$, $^{o}m^h$, etc. This makes it possible to actually read the draw-

ings without the aid of the text, and after the text has once been referred to, the value of the drawing, as a reminder, is materially enhanced. Fig. 43 is in the nature of a notation chart; the solution drawing, of which this is a special expansion, will be found at Fig. 94. Throughout, drawings carry such explanatory

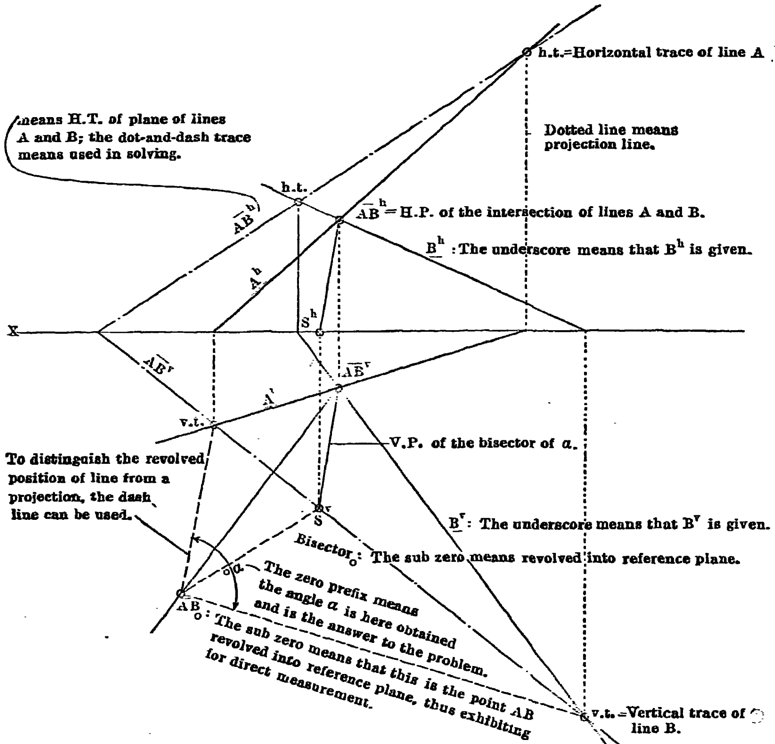


FIG. 43

By means of the notation, as illustrated in this chart, a descriptive drawing can be read without the aid of the text; thus in Fig. 94, of which this is an expansion, it is evident at once that the intersecting lines A and $\underline{B}$ are given, and that the solution exhibits α for direct measurement; also that the line $\overline{AB}$ s is the bisector of α.

notation and nomenclature as has been deemed necessary for clearness, and thus many wordy and tiresome explanations have been avoided.

The Reading of Descriptive Drawings: There are two ways in which the reading of a descriptive drawing differs from the reading of a mechanical drawing. In the first place, descriptive drawings deal with invisible geometrical magnitudes—points,

lines, and surfaces—and should be read without regard to any substance or solid body that these magnitudes may actually define. In the second place, while a mechanical drawing is read as specifying the shape, dimensions, and material of something that is to be manufactured, and depends for its meaning chiefly upon the shape illustrated by a group of projected views, a descriptive drawing should be read as the solution of a stated problem, and depends for its meaning chiefly upon the notation it carries, and only to a limited extent upon the arrangement of its lines. In brief we may define a mechanical drawing as an "illustrated specification," a descriptive drawing as a "solution of a problem."

In descriptive drawings there are practically no "constructions," and any attempt to memorize a solution by the appearance of a drawing, or what may appear to be a construction, without a full understanding of the problem which it solves and the analysis which it illustrates will completely fail.

It has been stated in Chapter I, third paragraph, that the analysis of a problem is distinctly more important than the example solution shown. Particular attention is directed to this statement, because a proper study of descriptive geometry, depending upon these analyses, not only provides the engineer with a means of solving problems of practical value, but develops an alertness and accuracy of the imaginative quality of mind which is of great value to him, no matter what branch of engineering work he may follow. Every engineer should know his descriptive. And the knowing depends upon the essentials, which can all be summed up under the relations of points, lines, and surfaces. Anything that will help the student to get hold of these relations is worth while; the special notation signs are of considerable assistance in the reading of solution drawings. Fig. 43 should be carefully studied before proceeding.

CHAPTER XII

LINE ESSENTIALS

IN order that we may be able to place a line in any required position in space, it is necessary to consider all its relations with H and V. We must remember that a line has not only *location* but also *inclination* and *direction.* Thus in Figs. 12 and 13 we see at once that the line ab has not only a certain location with reference to H and V, but also certain inclinations and a direction.

The location is expressed by the two projections; the inclinations are expressed by the angles that it makes with H and V; the direction is a visible quantity; thus a line making 30° with H may point up from the left or up from the right, etc.

Notation: The angles that a line makes with H and V are called θ and ϕ respectively. It is easy to remember which angle is θ and which is ϕ by the horizontal line in one and the vertical line in the other.

It is impossible to imagine a line in space that does not have the following four relations with H and V:

H.P. = horizontal projection.
V.P. = vertical projection.
θ = angle with H (anything from 0 to 90°).
ϕ = angle with V (anything from 0 to 90°).

These four relations are the "line essentials." Having any two of them, solutions can be found for the other two.

Problem 2: Given H.P. and V.P. to find θ and ϕ.

Analysis No. 1: The angle that a line makes with a reference plane is equal to the angle that it makes with its projection on that plane.

Solution: Given the line ab, Fig. 44. To find θ: Revolve the line into H, about axis a^hb^h; then the angle between a_ob_o and a^hb^h is θ, exhibited for direct measurement. To find ϕ: Revolve the line into V about axis a^vb^v; then the angle between a_ob_o and a^vb^v is ϕ. Note that the intersections of the revolved positions and the projections are the line traces, h.t. and v.t.

Analysis No. 2: The angle that a line makes with a reference plane is equal to the base angle of the right cone whose element of revolution is the line and whose base is in the given plane or parallel to it. Note: A right cone is one whose base is a circle in a plane at right angles to the axis. Throughout this book, the word "cone" means right cone.

This analysis is less direct than No. 1, but it is more interesting and more valuable, and should be thoroughly understood and appreciated at once. In Fig. 45 a reference plane is shown, with a line in space inclined to it. The line is represented as the element of a right cone with base in the reference plane. The base angle of the cone is marked. With the aid of this drawing the student should be able to understand the analysis.

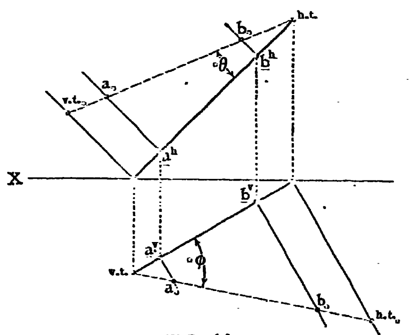


FIG. 44

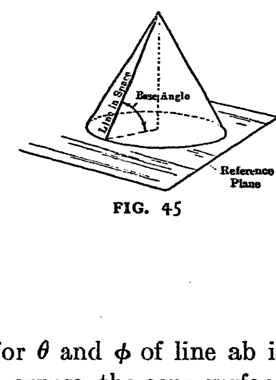


FIG. 45

Solution: In Fig. 46 the solution for θ and ϕ of line ab is given. In this drawing, for the sake of clearness, the cone surface is shaded.

Nomenclature: The cone whose base angle equals θ is called a "θ cone"; the cone whose base angle equals ϕ is called a "ϕ cone." In each case, in Fig. 46, the cone base is shown in an auxiliary reference plane—H_1 parallel to H, and V_1 parallel to V.

Referring to Fig. 46, the student should see that what we have really done in this solution is first to revolve the line ab, about vertical axis through b^h, until parallel to V; and that in doing so we have exhibited the true length of the line at $b^v a_o$, as well as the angle θ; then we have revolved the line about horizontal axis through b^v until parallel to H, and so have again exhibited the true length of the line at b^h a_o, as well as the angle ϕ.

It is by the complete revolution of the line—first about one axis, then the other, that the θ and ϕ cones are generated.

Fig. 47 is a model drawing made with the one purpose of helping the student to get the idea of a line in space as an element of its θ cone. The H and V projections of the line are shown; also a vertical axis aa^h passing through a point of the line. The revolution of the line about the axis generates the θ cone. The ϕ cone of the same line could be illustrated in a similar manner.

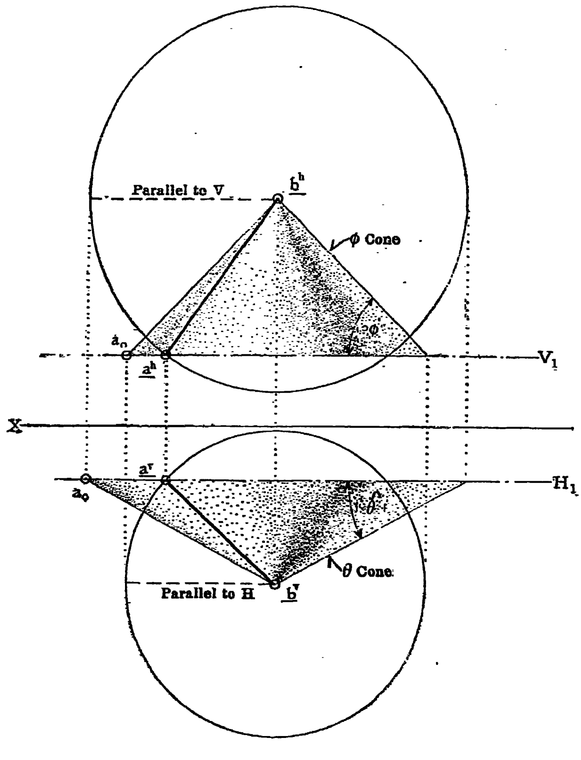


FIG. 46

Inclinations: $\theta + \phi$ (for lines) cannot be less than 0° or greater than 90°. If the line is parallel to X, $\theta + \phi = 0°$.

If the line is perpendicular to either H or V, either θ or $\phi = 90°$ and the other angle $= 0°$.

If the line is in a plane at right angles to X and inclined to both H and V, $\theta + \phi = 90°$.

For all other general positions of a line in space, $\theta + \phi$ is between 0° and 90°.

Problem 4: Given H.P. and θ, or V.P. and ϕ; to solve for the other projection. (Note that to "solve for" indicates that there is more than one solution.)

Analysis No. 1: From any point in the given projection draw a line making the given angle with it; consider this as the revolved position of the line itself; then by counter-revolving, the other projection is obtained.

Solution: In Fig. 48, C^h is given, and θ equals 45°; to find the V.P. From any point in C^h, as a^h, draw $a^h b_o$ making 45°

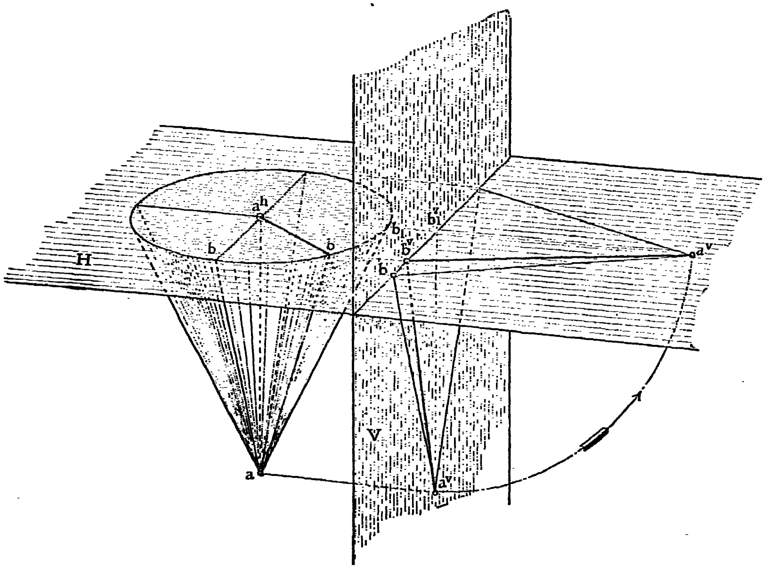


FIG. 47

with C^h. Counter-revolve b_o by making sb^v equal to $b_o b^h$; then $a^v b^v$ is one solution for C^v.

Other Solutions: Any line drawn parallel to $a^v b^v$ is another solution, and there are an infinite number of them. In this case the given essentials determine the *location* of the line with regard to V, and the *inclination* with regard to both H and V. The location with reference to H is not determined, and as the problem does not say anything about *direction*, that is a matter of choice.

Analysis No. 2: The line is an element of the cone of given base angle, having its base in the plane of the given projection, or parallel to it.

Solution: Given H.P., and $\theta = 60°$; to solve for the V.P.

Revolve the line about a vertical axis containing any point a, Fig. 49, in the given projection. With any radius, as $a^h b^h$, draw the θ cone base. Construct the V.P. of the θ cone; then by

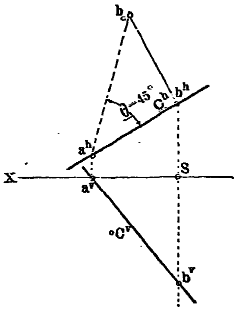


FIG. 48

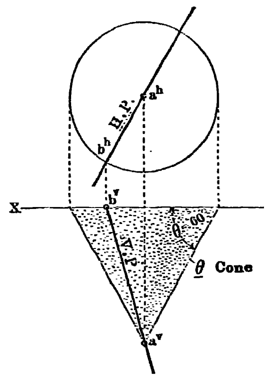


FIG. 49

simple projection the required V.P. of the line is at $a^v b^v$. The solution for one direction only is shown.

Problem 5: Given H.P. and ϕ; or V.P. and θ; to solve for the other projection.

Analysis: The line is an element of the cone of the given base angle, having its base in the plane of the required projection, or parallel to it.

Solution: Given H.P., and $\phi = 30°$, Fig. 50; to solve for the V.P.

Revolve the line about an axis perpendicular to V and containing any point in the line, as $b^h\ b^v$; b is the apex of the ϕ cone. Construct the cone with base angle = 30° and base in either V or V_1; then by simple projection the required V.P. is at $a^v b^v$. The solutions for two directions are shown.

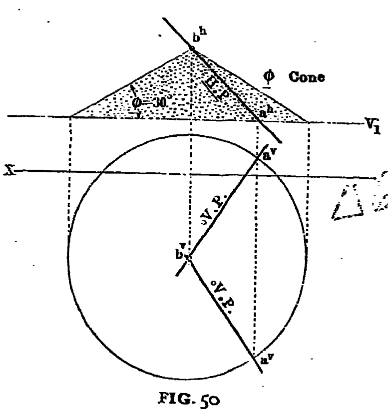


FIG. 50

Problem 6: Given θ and ϕ; to solve for H.P. and V.P.

Analysis: The line is that element of its θ cone which makes ϕ degrees with V; or, it is that element of its ϕ cone which makes θ degrees with H.

Referring to model drawing, Fig. 47, it is evident that if ab is revolved about its θ cone axis aa^h, its angle with H remains unchanged, and always equals θ; but its angle with V varies from a maximum at ab_1 (when ϕ = complement of θ) to zero at ab_o (when the line is parallel to V).

We also see that when ϕ is maximum and the element is at ab_1, the V.P. is of minimum length; and when $\phi = 0°$ (element parallel to V) the V.P. is its maximum, or true, length $a^v b_o$.

Between these two positions, ab_1 and ab_o, the V.P. of the element will have different lengths, depending upon the angle it makes with V.

Now if the angle ϕ is given, and the true length of the cone element is known (which of course it is) we have only to find

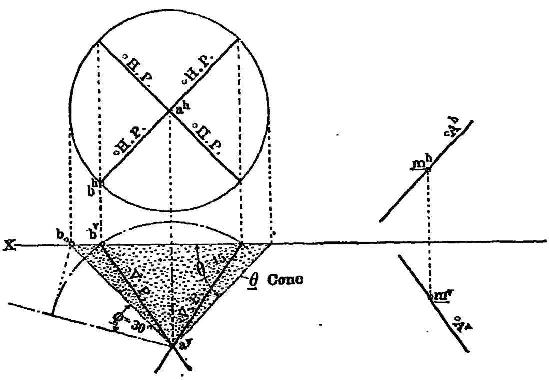


FIG. 51 FIG. 52

by construction what the length of its V.P. will be for the given angle ϕ, and to lay off this length from a^v to the V.P. of the cone base, and the required V.P. is located; then by simple projection the H.P. is at once obtained.

Solution: Given $\theta = 45°$, $\phi = 30°$; to find H.P. and V.P. The solution is shown in Fig. 51 which should be compared with model drawing, Fig. 47.

It is evident that there are four elements of the θ cone which make 30° with V, giving four different "direction solutions" to the line; they are all shown in Fig. 51.

Parallel Lines: If the like projections of two lines are parallel to one another, the lines themselves are parallel.

If we require that a line of certain inclinations and direction

shall contain a given point, we may first obtain the projections of a line in space, having the given inclinations and direction, and then through the given point draw projections parallel to those obtained.

Thus, if we wish to pass a line A through point m in Fig. 52, given $\theta = 45°$, and $\phi = 30°$; we may solve problem as in Fig. 51, and then through m^h and m^v draw projections A^h, A^v parallel to a^hb^h, a^vb^v.

CHAPTER XIII

PLANE ESSENTIALS

In general a plane in space has five essential relations with H and V, namely:

H.T. = horizontal trace.
V.T. = vertical trace.
θ = angle with H (anything from 0° to 90°).
ϕ = angle with V (anything from 0° to 90°).
K = true angle between traces (anything from 0° to 180°).

Certain planes—those that are parallel to either H or V—have only *three* essentials; one trace instead of two, and consequently no angle K between traces. Such planes may be termed special; as a rule a trace plane is thought of as having all five essentials.

If any two of the five essentials are given, solutions can be found for the other three; as with lines and their essentials, there is sometimes more than one solution from given data.

Inclinations: $\theta + \phi$ (for planes) cannot be less than 90° or greater than 180°. If a trace plane is parallel to one of the reference planes, $\theta + \phi = 90°$.

If a trace plane is parallel to X, $\theta + \phi = 90°$.

If a trace plane is perpendicular to X, each angle = 90°, and $\theta + \phi = 180°$.

If a trace plane is inclined to both H, V, and X, $\theta + \phi$ is between 90° and 180°.

General Principle: The solving of the more important plane-essential problems depends upon one simple principle, which may be stated thus: With relation to any reference plane, a trace plane can be considered as tangent to a right cone whose base lies in the reference plane.

This idea is illustrated in Fig. 53. The base angle of the tangent cone is the angle that the trace plane makes with the reference plane.

We are already familiar with the use of θ and ϕ cones in the

solving of line-essential problems; they are equally valuable in connection with the plane essentials.

Problem 7: Given H.T. and V.T.; to find θ and ϕ.

Analysis: θ is the base angle of the right tangent cone whose base is in H, or parallel to it; called the "θ cone." ϕ is the base angle of the right tangent cone whose base is in V or parallel to it; called the "ϕ cone."

The model drawing, Fig. 54, shows a θ cone of a trace plane.

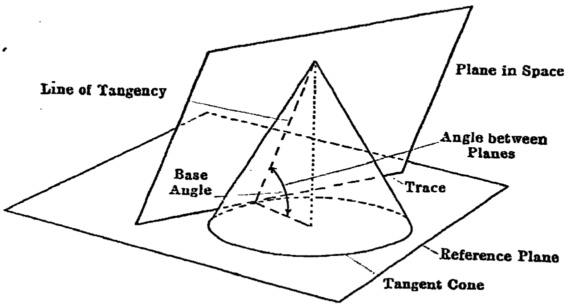


FIG. 53

If we analyze the conditions of tangency as exhibited in this drawing, we shall see that

1. The plane contains the apex of the cone.
2. The H trace is tangent to the base circle.
3. The straight line joining the apex and the point b of tangency of the H.T. is an element of the cone and lies in the trace plane.
4. The plane traces contain the like traces of the line; b is the h.t. of the line.

In the same figure there is shown a half cone, axis c in V; this half cone is seen to be tangent to the trace plane. As the axis is in V the apex c is in the V.T. Either position of cone can be used to exhibit θ for direct measurement; but the second one, the half cone, is generally used because it requires fewer lines to draw it. Fig. 55 is lettered the same as Fig. 54, and should be carefully compared with it; the element of tangency cd is a line of the plane; and the base angle θ is exhibited for direct measurement.

Problem 8: Given H.T. and V.T., to find K.

Analysis: Revolve one trace about the other as an axis into the plane of the axis trace; K will then be exhibited for direct measurement.

The model drawing Fig. 54 illustrates this analysis, showing the first-angle portion of the trace plane revolved into H about the H trace.

We have seen that in order to handle a line we must use two points in it. Referring to the model drawing and also to the descriptive drawing Fig. 56, which means the same: In order to revolve 3^v about 3^h as an axis, we must have the projections of two points in 3^v, as a and s. The point a has projections a^h, a^v;

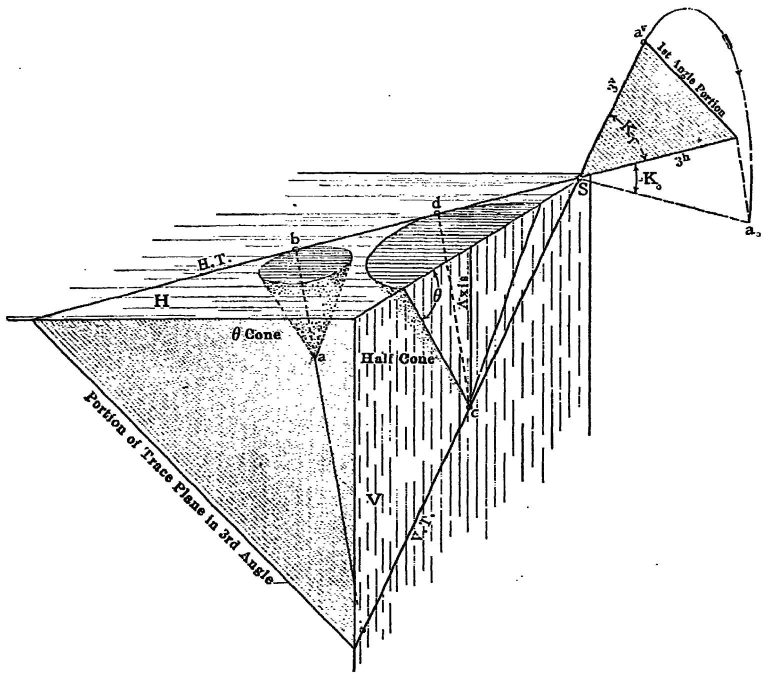


FIG. 54

the revolved position of a is marked a_o. The point s is in the axis trace 3^h, and therefore does not move; so that sa_o is the revolved position of 3^v (note that $sa_o = sa^v$) and the angle $3^h sa_o$ is the required angle K.

In connection with this problem it is as well to notice that a plane of intersecting traces is cut into four "dihedral portions" by its traces, and that there are therefore four K angles, which we may name, after their dihedral angles, K_1, K_2, K_3, K_4. Evidently $K_1 = K_3$; $K_2 = K_4$; also, adjacent K angles are supple-

mentary; so that if one is determined, the opposite one is equal to it and the other two are supplementary.

Problem 9: Given one trace and the angle with the plane of the same trace; to solve for the other trace.

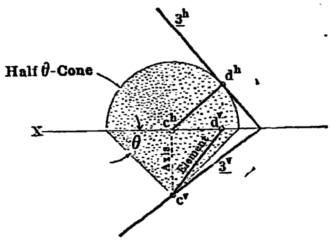


FIG. 55

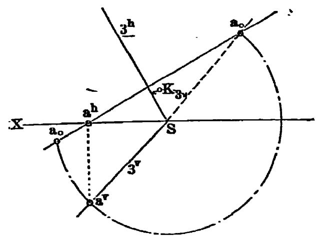


FIG. 56

Analysis: The other trace will intersect X in the given trace, and will contain the apex of the cone whose base is in the plane of the given trace.

For example solution see Fig. 57.

Problem 10: Given one trace and the angle with the plane of the other trace; to solve for the other trace.

Analysis: The other trace will intersect X in the given trace, and will be tangent to the base of the cone whose apex is in the given trace.

For example solution see Fig. 58.

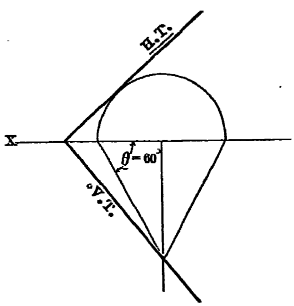


FIG. 57

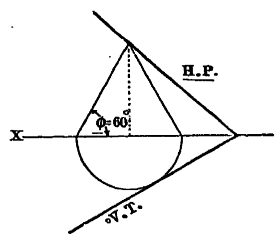


FIG. 58

Problem 11: Given one trace and the angle K between the traces, to find the other trace.

Analysis: In the reference plane of the given trace, the revolved position of the required trace can be drawn making angle K with given trace; by counter-revolving into the other reference plane, its position as the other trace is determined.

For example solution see Fig. 59; note that $sm^h = sm_o$.

Problem 12: Given K and either θ or ϕ, to solve for both traces.

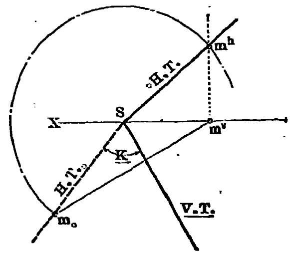


FIG. 59

Analysis: This problem is best analyzed in connection with model drawing Fig. 60, in which K and θ are given.

After drawing a θ cone with apex in V and base in H we see by inspection that three elements of the triangle abs are known:

The side ab; this is the tangent element.

The angle abs; we know that this is 90°.

The angle asb; this is the given angle K.

By construction, therefore, the triangle abs can be drawn, and the unknown side sa obtained; knowing that s is in X, and

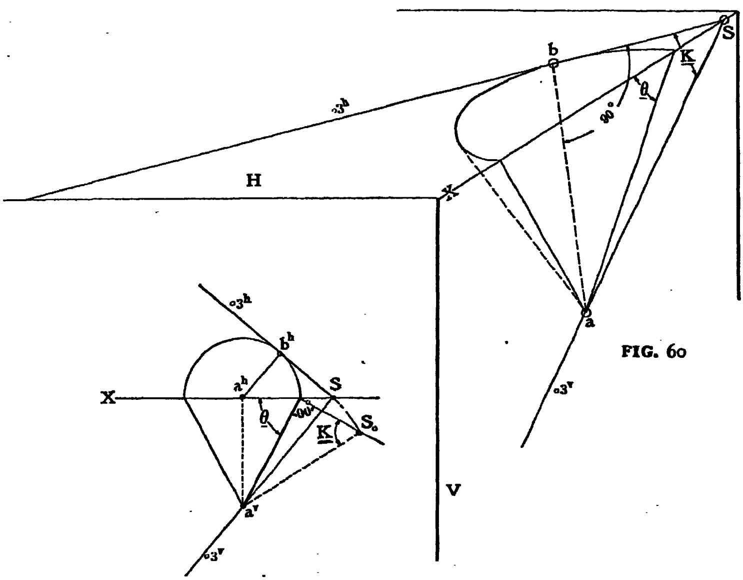


FIG. 60

FIG. 61

that sb is tangent to cone base, both traees can be drawn. Fig. 61 is the descriptive drawing of this solution.

Problem 13: Given θ and ϕ; to solve for both traces.

Analysis: The plane will be tangent to θ and ϕ cones which are themselves tangent to the same sphere whose center is in X.

A clear conception of this analysis can be obtained in the following manner:

1. Think of the required plane as tangent to a sphere whose center is in X. By rolling the plane over the sphere, it can be made to take any position with relation to the reference planes.

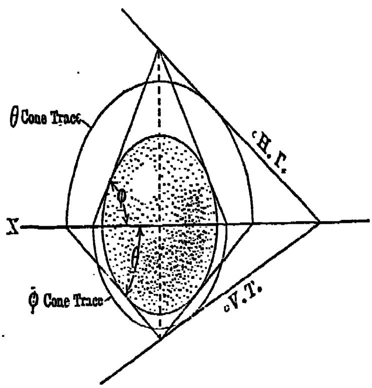


FIG. 62

2. Think of a tangent θ cone enveloping the sphere: The plane can be rolled over the sphere until it is tangent to this θ cone.

3. Think of a tangent ϕ cone enveloping the same sphere, its axis at right angles to that of the θ cone.

4. Now roll the plane over the θ cone until it is tangent also to the ϕ cone, and we have the plane in the required position.

5. The required H trace will be tangent to the base of the θ cone and will contain the apex of the ϕ cone; the V trace will be tangent to the base of the ϕ cone and will contain the apex of the θ cone.

For example solution, see Fig. 62.

Nomenclature: The line of intersection of a cone and a reference plane is called a trace of the cone. This nomenclature is used in Fig. 62.

CHAPTER XIV

DIRECT RELATIONS OF POINTS, LINES, AND PLANES IN SPACE

THERE are four analyses that have such wide application in the solution of problems concerning the direct relations of points, lines, and planes, that they may with advantage be taken as a group.

Four Analyses: 1. A line contains a point if the projections of the line contain the like projections of the point.

2. A line lies in a trace plane if the line traces are in the like plane traces.

3. A point lies in a plane if it lies in a line of the plane.

4. A line is perpendicular to a trace plane if its projections are at right angles to the like plane traces.

These four analyses, together with the converse of each, are all that are necessary for the solution of the following problems:

Problem 14: To pass a line through point m, Fig. 63.

Solution: The line A contains the point m because the projections of the line contain the like projections of the point.

Note: An infinite number of lines can be drawn through a point.

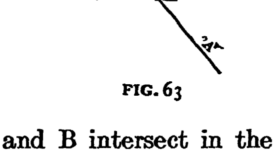


FIG. 63

Problem 15: To draw two lines that intersect.

Solution: In Fig. 64 the two lines A and B intersect in the point $\overline{AB}$ because the projections of the point are in the like projections of both lines.

Problem 16: To draw a line in the trace plane 3, Fig. 65.

Solution: The line A lies in the trace plane 3 because the h.t. of the line is in the H.T. of the plane and the v.t. of the line is in the V.T. of the plane. For the same reason B is in 3; note that A and B intersect.

Note: An infinite number of lines can be drawn in a given trace plane.

Problem 17: To show a point in the trace plane 4, Fig. 66.

Solution: The point m is in the trace plane 4 because it lies in A which is a line of the plane.

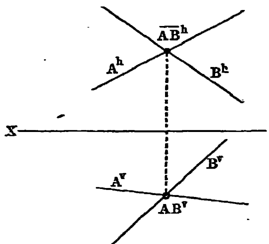


FIG. 64

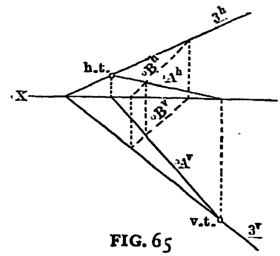


FIG. 65

Note: A point in either trace, as b in 4^h, is of course in the plane.

Problem 18: To pass a plane through line B, Fig. 67.

Solution: The plane 6 contains, or "passes through," line B because its traces contain the like traces of the line.

Note: An infinite number of planes can be passed through a line, as plane 7, etc.

Problem 19: To pass a plane through point b, Fig. 68.

Solution: The line C contains point b; the plane 2 contains line C; therefore, the plane 2 contains the point b.

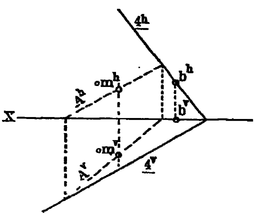


FIG. 66

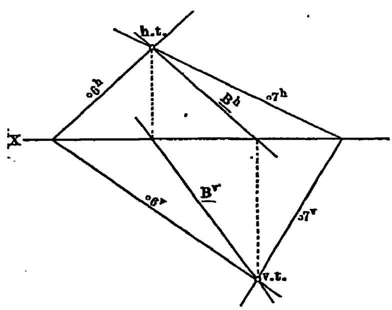


FIG. 67

Note: An infinite number of planes can be passed through a point.

Problem 20: Having given one projection A^h of line A in trace plane 3, Fig. 69; to find the other projection.

Solution: The h.t. of the given line is lettered h.t.^{h}, h.t.^{v}; the v.t. is lettered v.t.^{v}, v.t.^{h}; then the line joining v.t.^{v}, and h.t.^{v} is the required V.P. of line A.

Problem 21: To pass a plane through the two intersecting lines A and B, Fig. 70.

Solution: The traces of the plane $\overline{AB}$ contain the like traces of both A and B; therefore the plane contains both lines, and is said to pass through them.

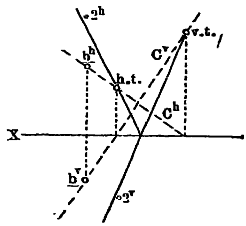


FIG. 68

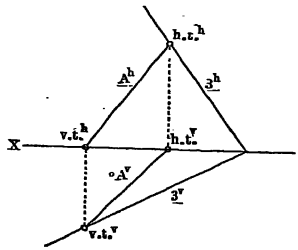


FIG. 69

Note: Two intersecting lines in space may be said to "determine" a plane.

Problem 22: To pass a plane through point c and line B, Fig. 71.

Solution: Through c draw a line to any point m in B; we now have two intersecting lines; the trace plane $\overline{cB}$ is the plane of these two lines and therefore contains c and B.

Note: A point and a line determine a plane.

Problem 23: To pass a plane through three points a, b, and c, Fig. 72.

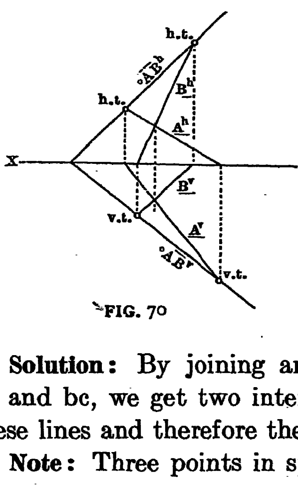


FIG. 70

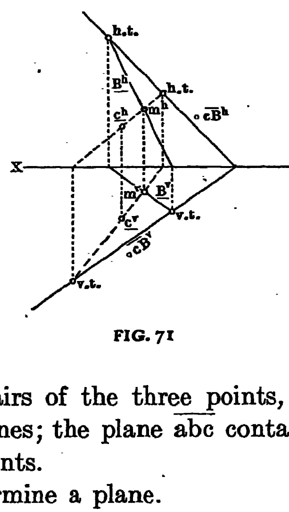


FIG. 71

Solution: By joining any two pairs of the three points, as ab and bc, we get two intersecting lines; the plane $\overline{abc}$ contains these lines and therefore the three points.

Note: Three points in space determine a plane.

Problem 24: To pass a plane through line A and parallel to line B, Fig. 73.

Solution: Through any point m in line A, draw an auxiliary

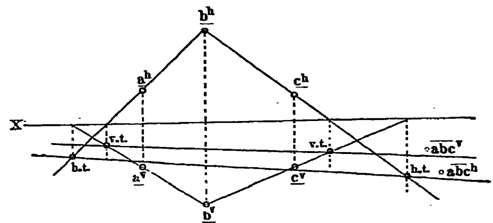


FIG. 72

line C parallel to B; plane $\overline{AC}$ contains these intersecting lines and is the required plane.

Problem 25: To draw a line perpendicular to the trace plane, Fig. 74.

Solution: The line A is perpendicular to the plane 4, because its projections are at right angles to the like traces of the plane.

Problem 26: To pass a plane through point m, Fig. 75, and perpendicular to the two trace planes 4 and 5.

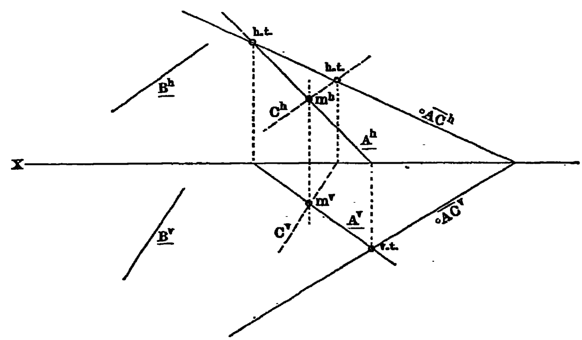


FIG. 73

Solution: Through m draw A and B perpendicular to the respective planes; the plane $\overline{AB}$ contains both these lines and is therefore perpendicular to both planes.

Problem 27: To pass a plane through line A, Fig. 76, and perpendicular to trace plane 7.

Solution: From any point m in line A, draw line B perpendicular to given plane; the plane $\overline{AB}$ is the one required.

Problem 28: To place a line in a given trace plane 3, Fig. 77, so that it shall make θ degrees with H.

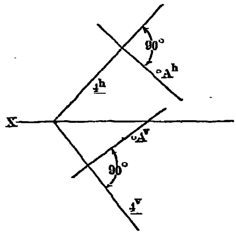


FIG. 74

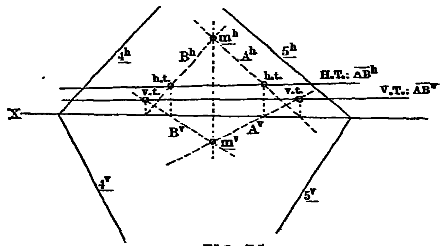


FIG. 75

Condition: Line θ cannot be greater than plane θ.

Solution: Taking line θ less than plane θ: With any point m in the plane as apex, construct a θ cone for the required line;

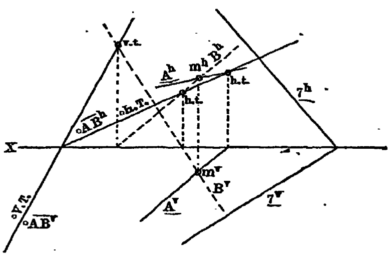


FIG. 76

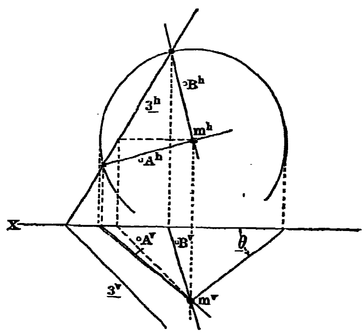


FIG. 77

the base of this cone, drawn in H, will intersect 3^h in two points, from which two elements of the cone can be drawn, which will lie in the plane 3. There are thus two solutions.

CHAPTER XV

INDIRECT RELATIONS

THE solutions of the problems given below are much less direct than those given in the previous chapter, and though finally dependent upon the same group of analyses, they are decidedly more difficult to comprehend. For these reasons they are separated from the others and classed as "indirect." They are of vital importance in both the study and the application of descriptive geometry, and should be mastered at once.

General Analysis: If two planes are parallel, and a line is placed in one of them, a parallel line drawn through any point

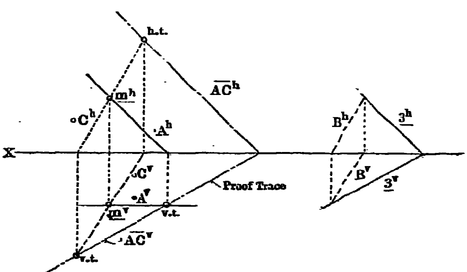


FIG. 78

in the second plane will lie *in* the second plane, and will therefore have its traces in the like traces of the second plane.

Problem 29: To draw a line through m, Fig. 78, so that it shall lie in a plane parallel to the given plane 3.

Solution: Remembering that a trace of a plane is a line of the plane, we draw through m a line A parallel to the line 3^h; this line A will lie in a plane whose traces are parallel to those of the given plane 3.

Any other line, as B, in plane 3 will answer the same purpose as the line 3^h which we used; thus the line C through m parallel to B will also lie in a plane parallel to 3.

We now have two lines which by analysis are in a plane parallel to plane 3; solving for the plane of A and C and finding their traces parallel to 3^h and 3^v is proof.

Problem 30: To pass a plane through point m and parallel to plane 3, Fig. 79.

Solution: Through m draw a line A parallel to either trace of plane 3, as 3^v; this line A has only a horizontal trace, but this is enough to determine the required plane; for we know the directions of both its traces and a point in one; through the h.t. of line A draw the H. T. of the plane parallel to 3^h, and through the intersection s of the H.T. and X draw the required V.T. parallel to 3^v.

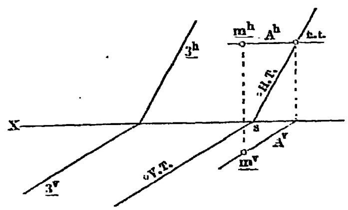


FIG. 79

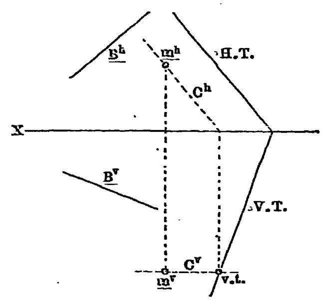


FIG. 80

Problem 31: To pass a plane through given point m, Fig. 80, and perpendicular to given line B.

Solution: The traces of the required plane will be at right angles to the like projections of the given line B; through m draw line C parallel to the H.T. of the plane; C^h will be at right angles to B^h, and C^v will be parallel to X; the plane of this line, with traces drawn at right angles to the like projections B^h and B^v, is the required plane.

CHAPTER XVI

INTERSECTIONS OF LINES AND PLANES

Two planes that are not parallel intersect in a straight line; to determine the line, which we call the intersection, it is only necessary to join any two points that are common to both planes.

Problem 32: In Fig. 81, planes 2 and 3 are given; to find their intersection.

Solution: The point where 2^h intersects 3^h is common to both planes. The point where 2^v intersects 3^v is common to both planes. The line joining these points, whose other projections are of course in X, is the required intersection $\overline{23}$.

Different Cases: Solutions of different cases are given in Figs. 82 to 91; they are all true to the same analysis, but the solutions in detail depend upon the relative positions of the traces themselves.

When, as in Figs. 82 to 86, the like traces meet within the limits of the drawing, the solution given in Fig. 81 has direct application.

When one pair of like traces are parallel, as in Fig. 87, only one point in the required line of intersection is found, but as we know that the intersection is parallel to the parallel traces, this one point is enough.

When both pairs of traces are parallel, as in Fig. 88, or when the traces intersect in X, as in Fig. 89, the intersection is obtained by finding the point of intersection of their profile traces.

When, as in Fig. 90, two like traces are so nearly parallel or so far apart on the drawing that they do not meet within the limits of the board, an auxiliary reference plane is used.

Explanatory: The model and descriptive drawings in Fig. 93 explain the use of an auxiliary reference plane: A trace plane 3 is shown; the auxiliary plane V_1 intersects plane 3 in a line that is necessarily parallel to 3^v, because two parallel planes intersect a third plane in parallel lines. Any plane parallel to V can be used as an auxiliary vertical plane and will intersect any trace plane in

a line parallel to the V.T. of that plane; similarly every plane parallel to H will intersect the trace plane in a line parallel to the H.T. of that plane.

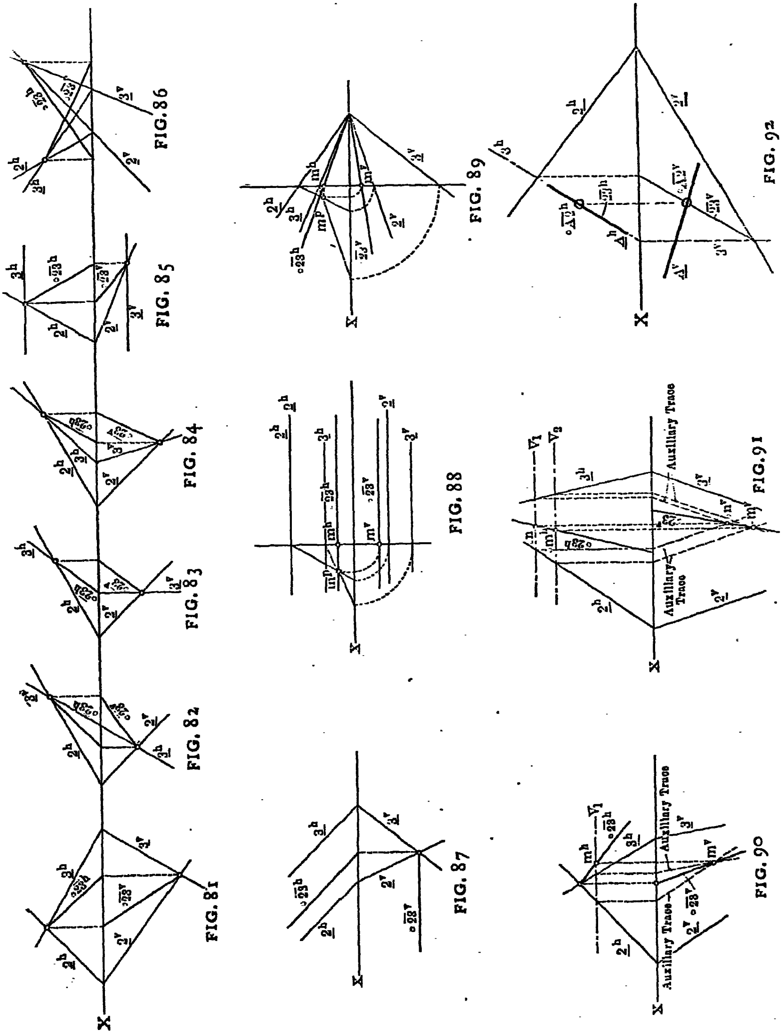


FIG. 81, FIG. 82, FIG. 83, FIG. 84, FIG. 85, FIG. 86, FIG. 87, FIG. 88, FIG. 89, FIG. 90, FIG. 91, FIG. 92

The intersection of any auxiliary reference plane with any given trace plane is called an "auxiliary trace."

In the solution shown in Fig. 90, V_1 gives a pair of auxiliary traces of the given planes, and their point of intersection m is in

the required line of intersection. The point where 2^h and 3^h intersect is another point, and thus the line 23 is determined.

When, as in Fig. 91, neither pair of traces meet within the limits of the board, *two* auxiliary reference planes are used.

Problem 33: To find the point of intersection of a line and a trace plane.

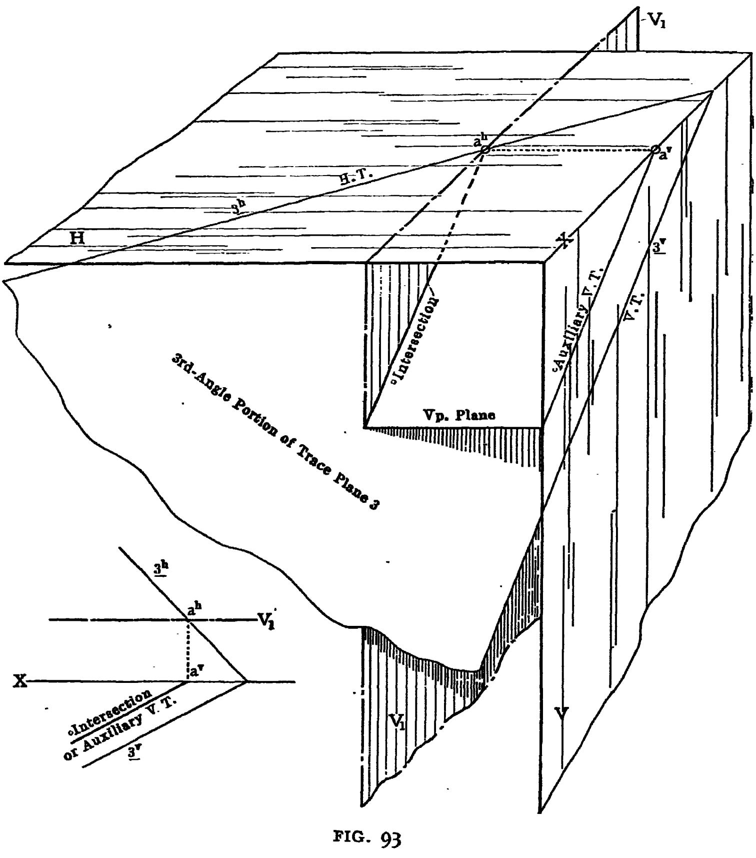


FIG. 93

Analysis: The point is where the given line meets the line of intersection of the given plane and any auxiliary plane containing the given line.

Solution: In Fig. 92 the auxiliary plane 3 is the Hp plane of the given line. It is generally convenient to use either one or other of the projector planes in this way, giving a direct solution with the smallest number of lines.

CHAPTER XVII

ANGLES BETWEEN INTERSECTING LINES AND PLANES

PROBLEMS relating to the placing of lines and planes so that they shall make required angles with the reference planes have been discussed in chapters XII and XIII. This chapter deals with the angles between lines intersecting in space, between trace planes, and between lines and trace planes.

Notation:

α (alpha) = angle between two intersecting lines.

δ (delta) = angle between two trace planes.

Σ (sigma) = angle between a line and a trace plane.

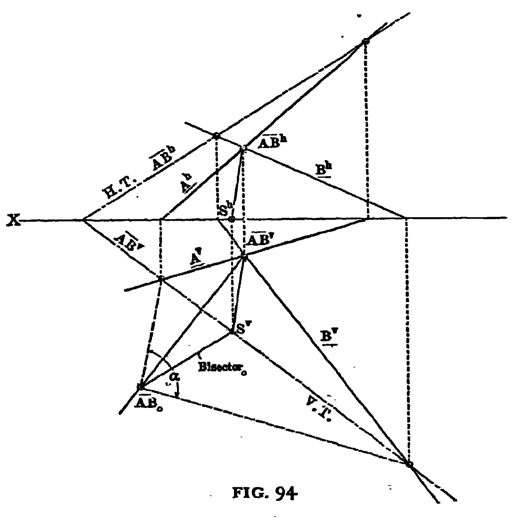


FIG. 94

Exhibiting for Direct Measurement: Solutions for true angles furnish a direct application of the revolution about an axis, by which means angles are revolved into one or other of the reference planes and are thus exhibited for direct measurement.

Problem 34: To find the angle α between two intersecting lines. There are three analyses:

Analysis No. 1: Revolve the plane of the lines (carrying the lines with it) into either H or V, and a will be exhibited for direct measurement (see Fig. 94).

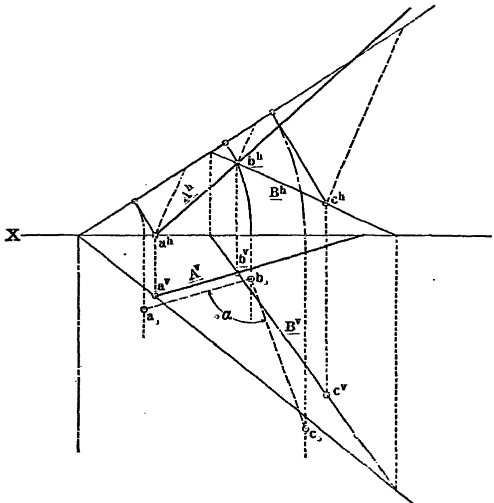


FIG. 95

Analysis No. 2: Revolve the plane of the lines about one trace until perpendicular to the reference plane of that trace; then revolve it about its new trace—which is perpendicular to X—into the reference plane of that trace, and a will be exhibited (see Fig. 95).

Analysis No. 3: Intersect the given lines by a third line, so as to form a triangle; find the true length of each side and construct the triangle, thus obtaining a (see Fig. 96).

Problem 35: To find the angle δ between two given planes. There are two analyses:

Analysis No. 1: Pass an auxiliary plane perpendicular to the given planes and find its two lines of intersection; the angle between these lines is δ, and can be exhibited by any one of the three analyses given (see Fig. 97).

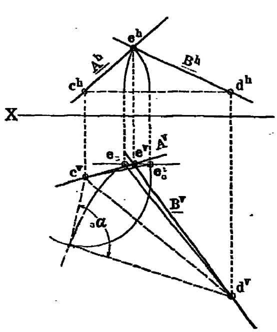


FIG. 96

Analysis No. 2: From any point not in either plane, as m, Fig. 98, draw two lines perpendicular to the two planes,

and find their angle α; the supplement of this α is the required angle δ.

Problem 36: To find the angle Σ between a given line and a given trace plane.

Analysis: From any point in the line draw a line perpendicular

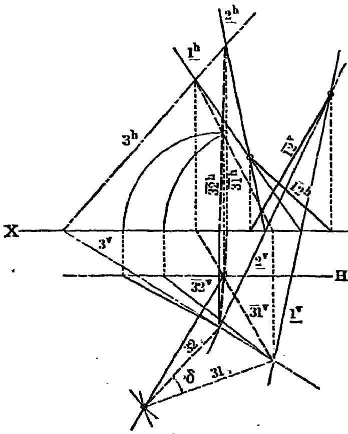


FIG. 97

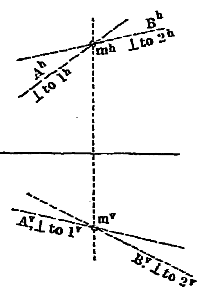


FIG. 98

to the given plane. The α of the two lines is the complement of the required Σ (see Fig. 99).

Problem 37: To pass a plane through a given line so that it shall make angle θ with H or ϕ with V.

Analysis: When θ is given: The plane traces will contain the line traces and the H.T. will be tangent to the base of a θ cone having apex in given line and base in H. There are two solutions, one plane on each side of the cone (see Fig. 100).

For analysis when ϕ is given substitute ϕ for θ and V for H in above.

Problem 38: To pass a plane making a given angle δ with a given trace plane.

Analysis: Anywhere upon the trace plane construct a cone having base angle δ and axis perpendicular to the trace plane; any plane tangent to this δ cone is a solution (see Fig. 101).

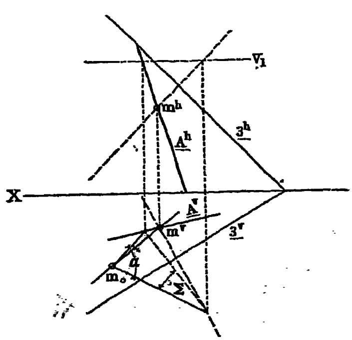


FIG. 99

Problem 39: To bisect the angle between two intersecting lines.

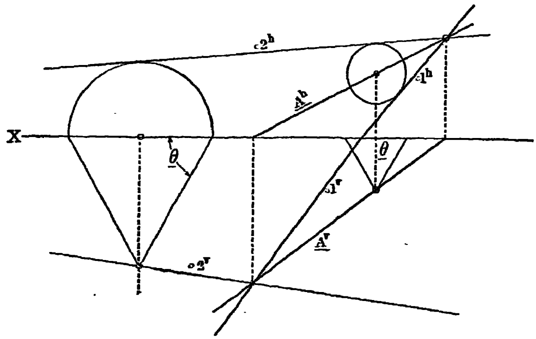


FIG. 100

Analysis: Revolve the lines into one of the reference planes; draw the bisector, and by counter-revolving, carry the bisector into position (see Fig. 94).

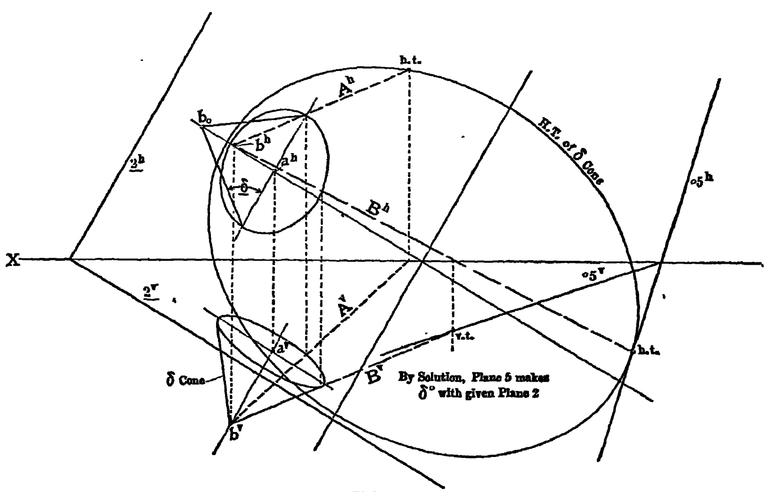


FIG. 101

CHAPTER XVIII

DISTANCES BETWEEN POINTS, LINES, AND PLANES

Nomenclature: In every case the word distance means "shortest distance."

Quite an important use of descriptive geometry is the finding of the shortest distance between two magnitudes in space, as between:

1. Two points.
2. Point and plane.
3. Two parallel planes.
4. Point and line.
5. Two lines not in the same plane.

Of these five problems, the last four resolve themselves finally into the first; in other words, the last step in each is to find the distance between two points in space. The real problem, however, in all except the first, is *to locate the two points*, and then, by revolving into one of the reference planes, to exhibit for direct measurement the distance between them.

Problem 40: To find the distance between two points. There are two analyses:

Analysis No. 1: Revolve both points into one of the reference planes about an axis containing both projections, and the true distance will be exhibited for direct measurement (see Fig. 42).

Analysis No. 2: Consider the line joining the points as an element of either its θ or its ϕ cone, and find the length of the element.

Problem 41: To find the distance from a given point to a given trace plane.

Analysis: The shortest distance is the length of the perpendicular from the point to the trace plane.

Solution: Referring to Fig. 102, b and 2 are given; the line M contains b and is perpendicular to plane 2, intersecting it at

point $\overline{M2}$. The distance from b to $\overline{M2}$ is the shortest distance between b and the plane, and is exhibited for direct measurement.

Problem 42: To find the distance between two parallel planes.

Analysis: Any line perpendicular to the planes will intersect them in two points whose distance apart is the required distance.

Problem 43: To find the shortest distance between a point and a line.

Analysis: A plane through the point and perpendicular to the given line will intersect the line in a point; the distance of this

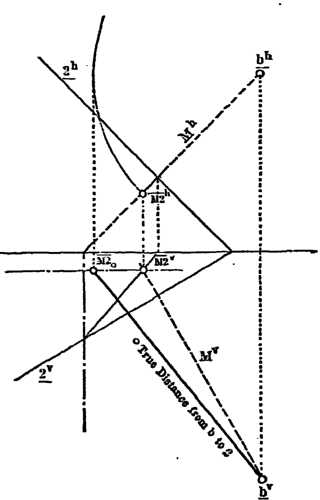


FIG. 102

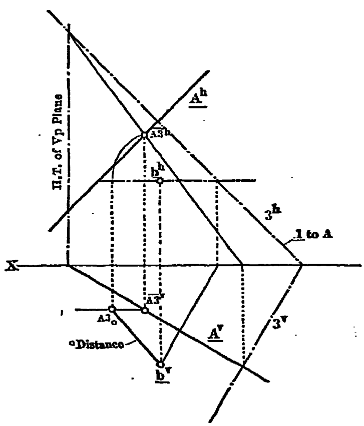


FIG. 103

intersection from the given point is the required distance. See Fig. 103.

Problem 44: To find the shortest distance between two lines that are not in the same plane. There are two analyses:

Analysis No. 1: The distance between parallel planes, one through each line, is the shortest distance between the lines: This analysis determines the distance but does not locate it.

Analysis No. 2: The following analysis, which both determines and locates the distance, is best given in connection with a solution:

Solution: Given lines A and B, Fig. 104; to locate and determine the shortest distance between them: Through A pass a plane

parallel to B; project B on to this trace plane; this projection will intersect the line A; from the point of intersection draw a line at

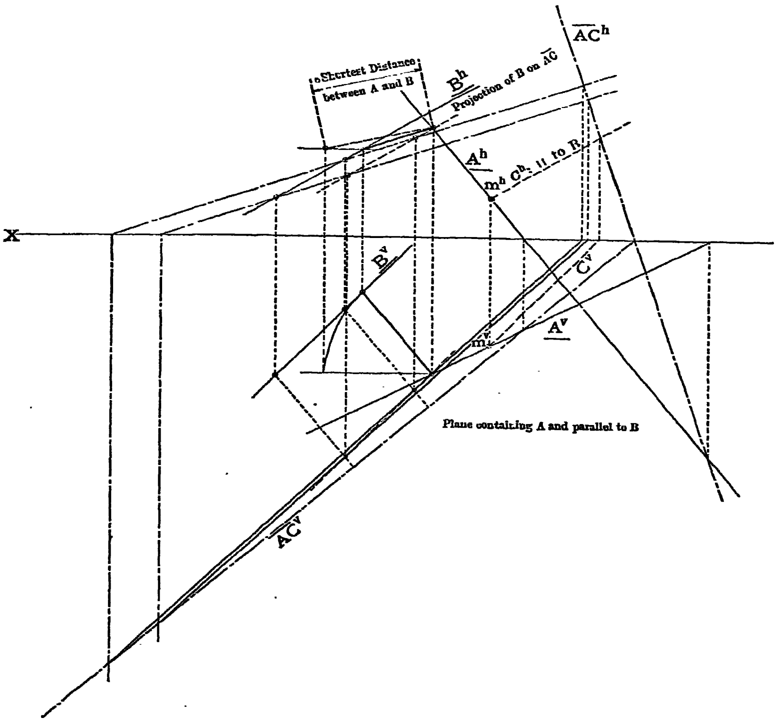


FIG. 104

right angles to the trace plane; this perpendicular will intersect the given line B, and both locates and determines the shortest distance between the lines.

CHAPTER XIX

CURVED SURFACES

CURVED surfaces are of several kinds:

If a line, either straight or curved, is revolved about an axis that is not at right angles to the plane of the line, it will generate a curved surface, called a surface of revolution.

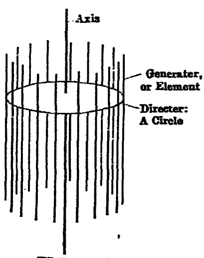


FIG. 105

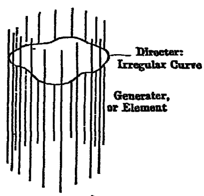


FIG. 106

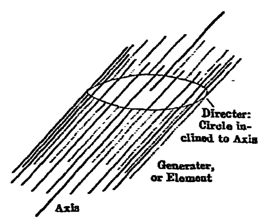


FIG. 107

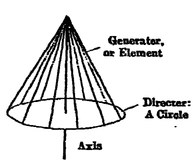


FIG. 108

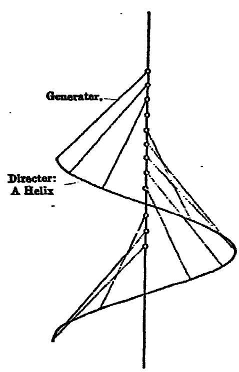


FIG. 111

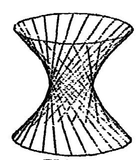


FIG. 112

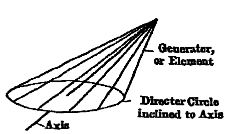


FIG. 109

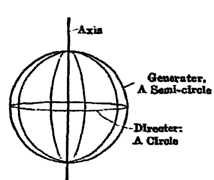


FIG. 110

The curved surfaces in most common use are of this kind, as the cylinder, the cone, and the sphere. Of these, the cylinder and cone are generated by revolving a straight line; the sphere by revolving a semicircle.

The line used to generate a surface is called the "generater" of the surface; any line that directs the motion of a generater is called a "directer."

Surfaces of revolution have circles as directers. A surface of revolution is called "right" if the plane of the directer is at right angles to the axis of revolution, and "oblique" if inclined to the axis.

A surface that has a straight-line generater is called a "ruled surface," and is such that through any point of the surface a straight edge can be so placed as to lie wholly in the surface. The simplest ruled surface is the plane.

Ruled surfaces other than surfaces of revolution have other curves as directers, either regular or irregular curves.

Figs. 105 to 112 illustrate these statements, and carry their own nomenclature.

CHAPTER XX

THE DEVELOPMENT OF SURFACES

All solid bodies are surrounded by their surfaces; these surfaces may be either plane or curved.

To develop a surface means to lay it out flat *without stretching any portion of it.* Evidently the surface of a cube, which consists of six planes, can be developed.

Curved surfaces are of two kinds: those that can be developed and those that cannot. Only ruled surfaces can be developed, and not all of these.

When any two consecutive elements of a ruled surface are in the same plane, the surface is developable. This condition is

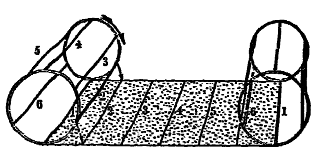


FIG. 113

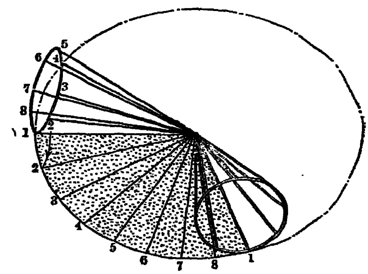


FIG. 114

fulfilled by all cylinders and cones, and by all other parallel-line and radial-line surfaces.

Neither the screw surface nor the hyperboloid are developable, because successive elements are not in the same plane.

Ruled surfaces that cannot be developed are called warped.

Surfaces generated by the motion of a curved line, as the sphere, can never be developed.

A common-sense conception of what is meant by laying a surface out flat is illustrated in connection with the cylinder and cone in Figs. 113 and 114. The curved surfaces, represented by

a few of their elements, are conceived as capable of making impressions of themselves on a plane; we may suppose the lines covered with printer's ink; then rolling the cylinder or cone over once, that is rolling it until it has made exactly one revolution, will print the development. The ordinary drafting process of developing a surface is the equivalent of this, and requires no special explanation.

CHAPTER XXI

PLANES TANGENT TO SURFACES OF REVOLUTION

WE are already accustomed to think of a trace plane as tangent to two cones. We have seen in connection with Fig. 54 what the conditions of tangency are. The same conditions hold for the cylinder except that there is no apex, the elements of a cylinder being parallel.

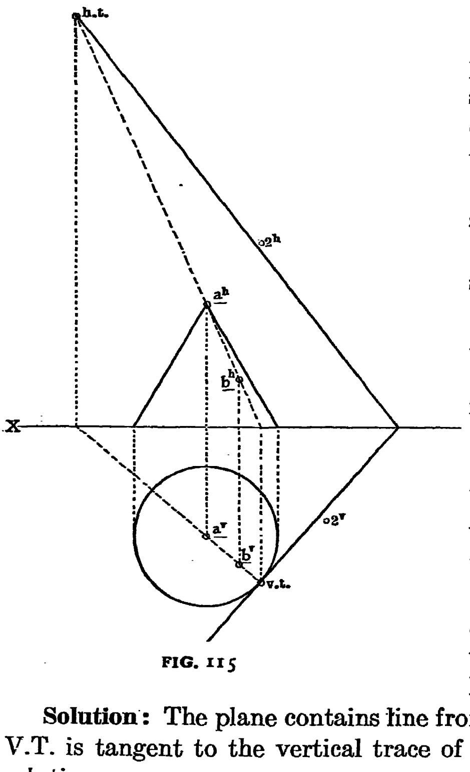


FIG. 115

General Analysis: A plane is tangent to a ruled surface of revolution if it contains one element of the surface and only one.

A plane is tangent to a curved-line surface if it contains one point of the surface and only one.

Problem 45: To pass a plane tangent to a right cone through a given surface point b, Fig. 115.

Solution: The plane 2 contains element through b, and the V.T. is tangent to the circle of base, or vertical trace of cone.

Problem 46: To pass a plane tangent to cone through outside point m, Fig. 116.

Solution: The plane contains line from m to cone apex, and the V.T. is tangent to the vertical trace of the cone. There are two solutions.

Problem 47: To pass a plane tangent to cone and parallel to given line B, Fig. 117.

Condition: Line must not intersect cone.

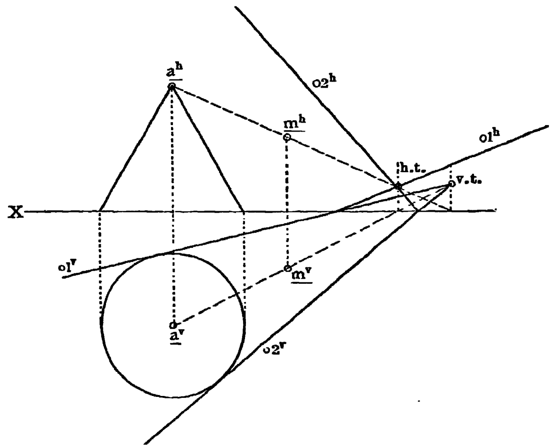


FIG. 116

Solution: Through the cone apex draw auxiliary line M parallel to B; find the traces of M; through the v.t. draw V.T. tangent to cone trace, which is in V; the H.T. contains the h.t. of the line

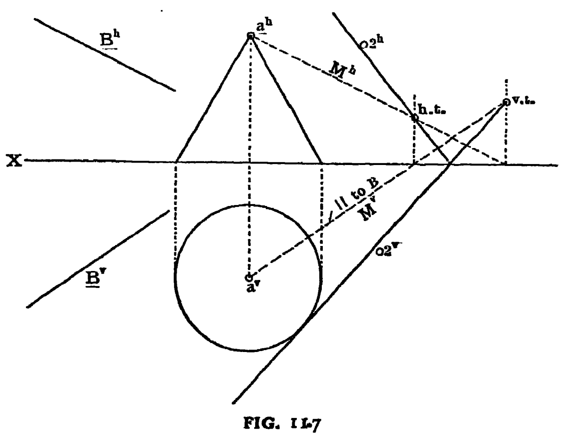


FIG. 117

and intersects X in the V.T. There are two solutions, one of which is shown.

Problem 48: To pass a plane tangent to oblique cone, Fig. 118, and making angle ϕ with the base, which is in V.

olution: Construct a ϕ cone with apex coincident with that ıe given cone; the required plane is tangent to both cones, .own, and there are two solutions, one shown.

Problem 49: To pass a plane tangent to an oblique cylinder, Fig. 119, and making angle ϕ with the base, which is in V.

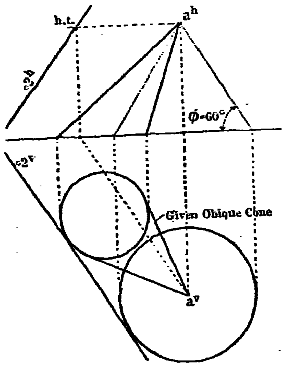


FIG. 118

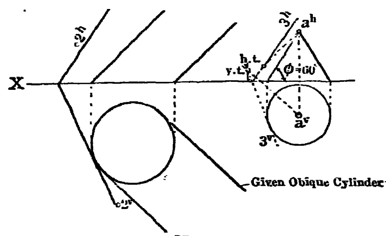


FIG. 119

Solution: Draw an auxiliary trace plane 3 making angle ϕ ı V and parallel to the axis or to any element of the cylinder; required tangent plane 2 is parallel to 3 and contains an element he cylinder.

Problem 50: To pass a plane tangent to sphere, Fig. 120, at a ın surface point m.

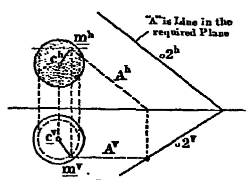


FIG. 120

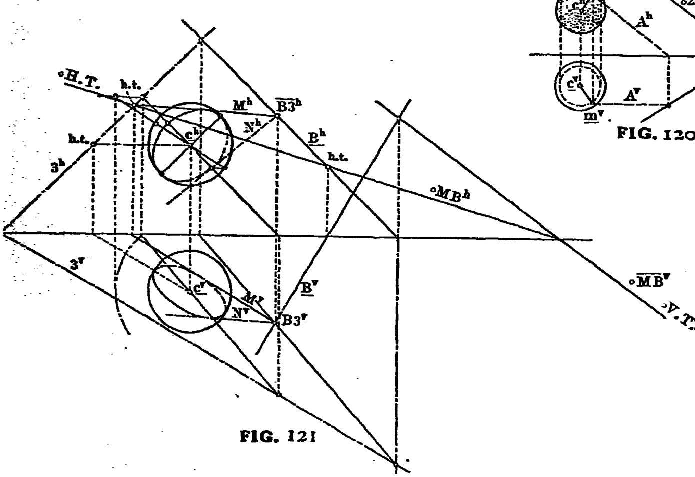


FIG. 121

Solution: Draw the radius of the sphere at m; the required gent plane is perpendicular to this radius at the surface point

m; in other words it contains m and is perpendicular to the radius through m.

Problem 51: To pass a plane tangent to sphere, Fig. 121, and containing given line B. There are two solutions:

Solution No. 1: Referring to Fig. 121, pass plane 3 perpendicular to line B and through the center of the sphere; this plane

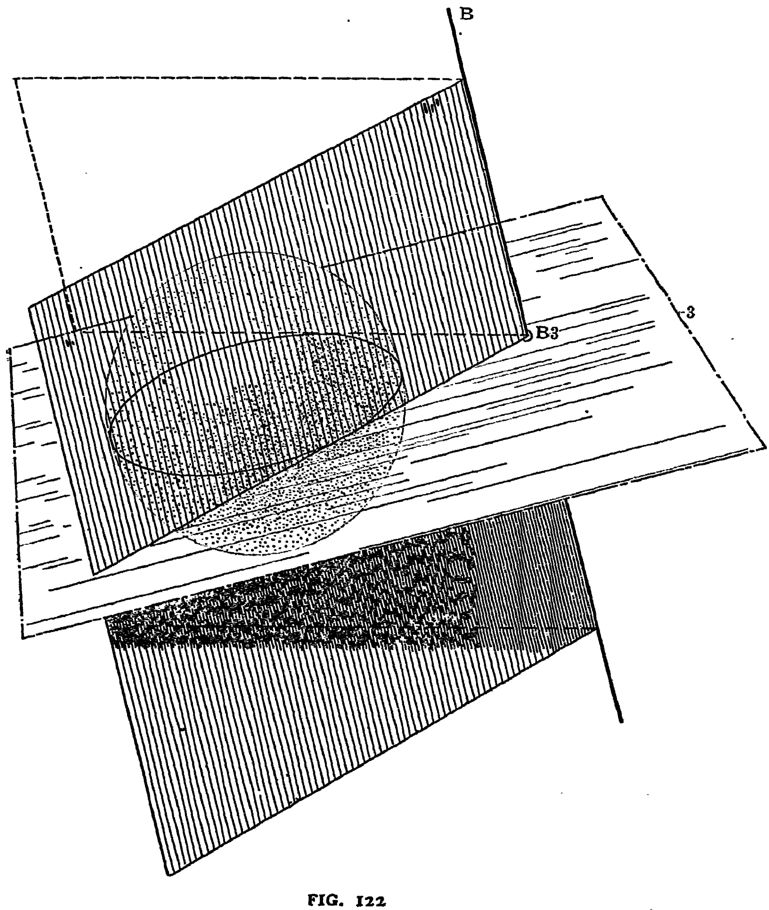


FIG. 122

cuts a great circle section of the sphere. From the point of intersection B3 of the given line and the cutting plane, two lines, M and N, can be drawn tangent to the great circle. The plane of B and either of these lines is a solution. Fig. 122 is a model

drawing of this solution; it means exactly the same as Fig. 121, and should be compared with it.

Solution No. 2: From any two points in line B draw two cones enveloping the sphere. The circles of tangency will intersect in two points; the plane of the given line and either of these points is a solution. This will provide an interesting exercise for the student.

CHAPTER XXII

PLANE SECTIONS OF SOLIDS

If we cut a solid with a plane we get a surface of division called a "section."

Taking the right cylinder as an example solid, there are three ways in which we can cut it, as illustrated in Fig. 123. The three sections we may call *longitudinal*, containing or parallel to the axis; *right*, perpendicular to the axis; and *oblique*, inclined to the

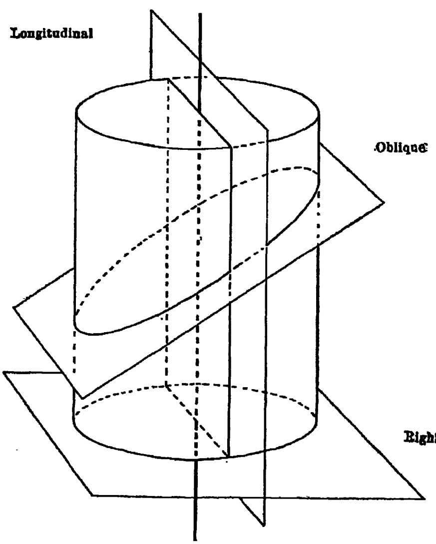


FIG. 123

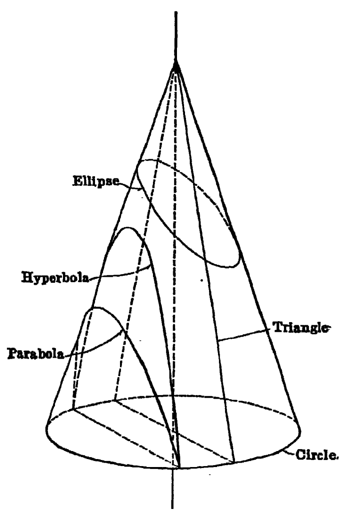


FIG. 124

axis. The longitudinal section is a rectangle; the right section a circle; and the oblique section an ellipse.

The cone has five sections: A triangle, if the cutting plane contains the apex; a circle, if perpendicular to the axis; an ellipse, if the angle with the axis is greater than that of the cone element; a parabola, if the angle with the axis is equal to that of

the element; and a hyperbola, if the angle is less than that of the element. These five sections are illustrated in Fig. 124.

The sphere has but one shape of section, namely circular.

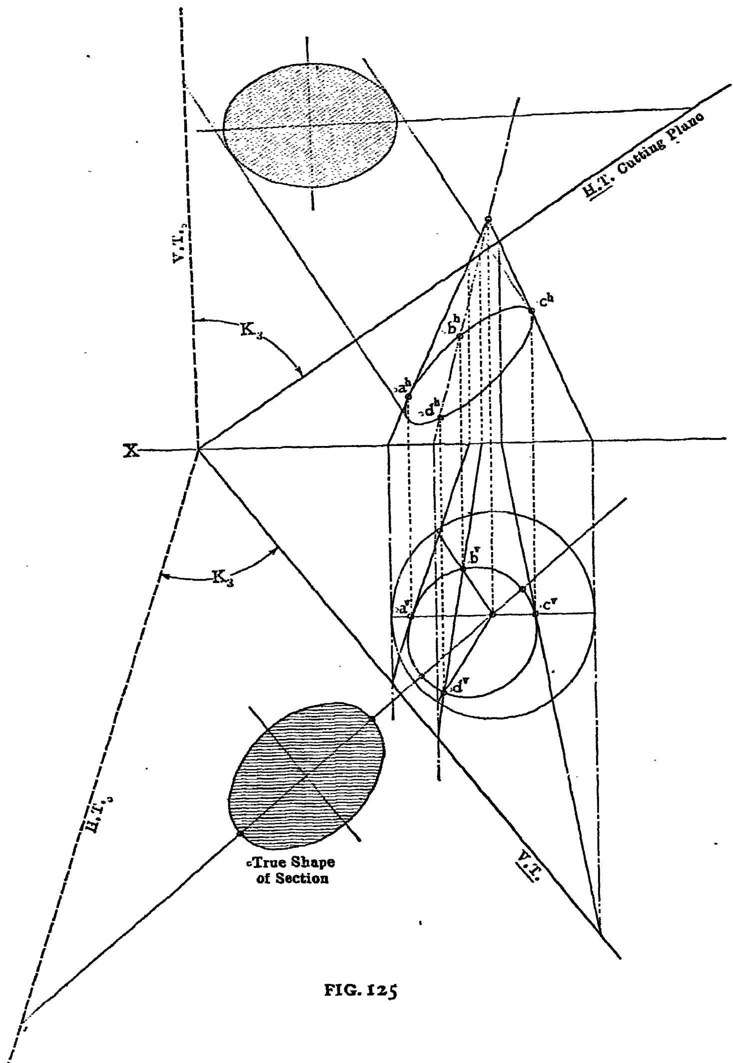


FIG. 125

Problem 52: To determine the shape of a section produced by cutting a solid with a plane.

Analysis: Every element of the surface will intersect the cutting plane in a point of the required section. If a sufficient

number of these points be found the line drawn through them will represent the shape of the section. The true shape of the section can be exhibited by revolving into one of the reference planes.

Solution: Fig. 125 shows the section of a given cone by a given cutting plane; the solution shows how a few points of the section were obtained.

In connection with all problems dealing with surfaces, a very useful conception of a line is a "succession of points of it"; and of a surface, a "succession of its elements." Then if we succeed in locating a sufficient number of the points of a line, we have located the line itself; and if we locate a sufficient number of the elements of a surface, we have represented the surface.

CHAPTER XXIII

INTERSECTIONS OF THE SURFACES OF SOLIDS

PROBLEMS on the intersections of surfaces call for considerable facility in handling the essentials of descriptive geometry.

It has already been suggested that a descriptive drawing should be read without regard to any solid substance that the geometrical magnitudes represented may seem to define. We are so accustomed when reading other kinds of drawings to think of the material objects shown that it is difficult at first to think only of the points, lines, and surfaces essential to the representation of these objects, without any regard whatever to any substance or solidity; and yet this is what we must do if we are to have the simplest and most direct conception of many problems. For instance, when we speak of a cone enveloping a sphere, we have in mind that the inner side of a cone surface is tangent to the outer side of a sphere surface, and that the line of contact is a circle; substance plays no part in this conception; the cone and the sphere are shells of infinite thinness and have no weight.

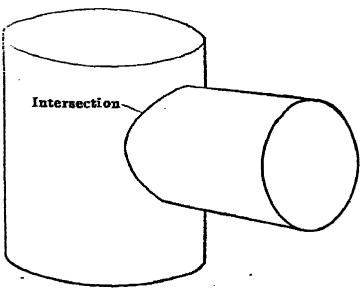


FIG. 126

In all intersection problems the so-called solid bodies, as cylinders, cones, spheres, prisms, and pyramids, are best regarded in this way. The vertical projection in Fig. 130 is shaded in accordance with this idea, showing the inner side of one cone surface and the outer side of the other.

In Fig. 126 two cylinders are shown intersecting. Their surfaces are seen to meet in a curved line, called their line of intersection, or simply "intersection."

If it were required to put together two such cylinders, made

of sheet metal, it would be necessary to determine their intersection, and the true shape or development of each ruled surface

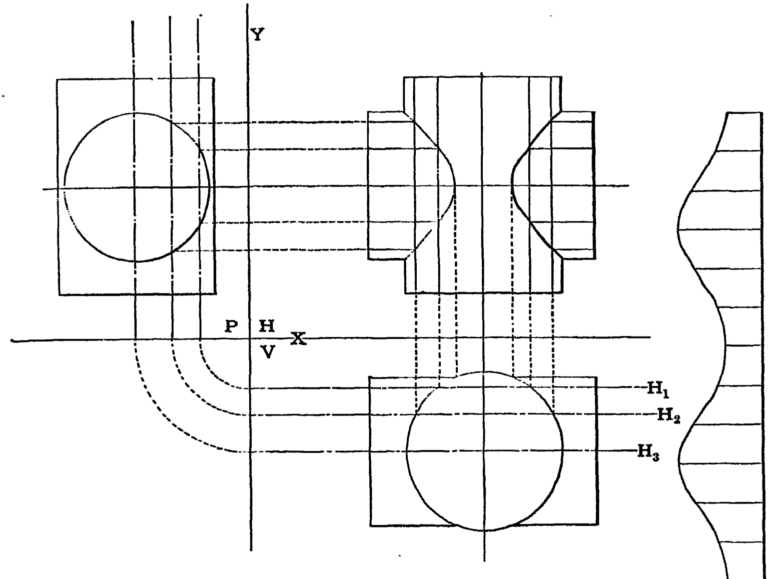


FIG. 127

up to the intersection. Then, flat sheets cut to the shape of these developments and rolled into cylinders would go together as shown.

To obtain the intersection, we must locate a sufficient number of the points of the line and draw a smooth curve through them.

Problems of this kind are readily solved by descriptive geometry. The method of solution is indicated in Fig. 127, and may be analyzed as follows:

Analysis: Through the intersecting surfaces pass a series of cutting planes containing elements of each surface; elements in the same cutting plane intersect in points in the required line.

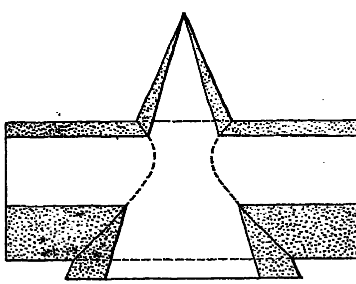


FIG. 128

Fig. 127 is a solution for two cylinders, their axes intersecting at right angles, and includes the development of the smaller cylinder.

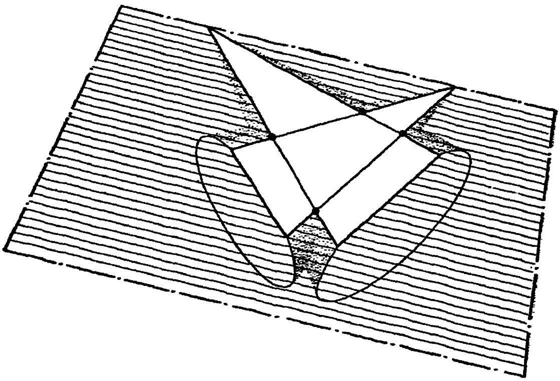


FIG. 129

Fig. 128 shows how the cutting planes must be used in the case of a cylinder intersecting a cone; each cutting plane must contain the apex of the cone and elements of the cylinder.

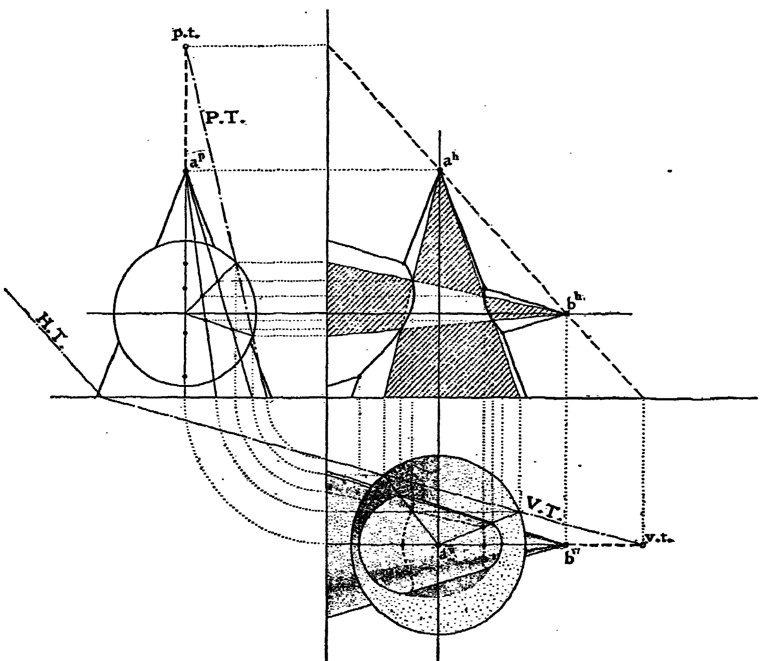


FIG. 130

Fig. 129 shows one of a series of cutting planes for obtaining the intersection of two cones, which problem is completely solved

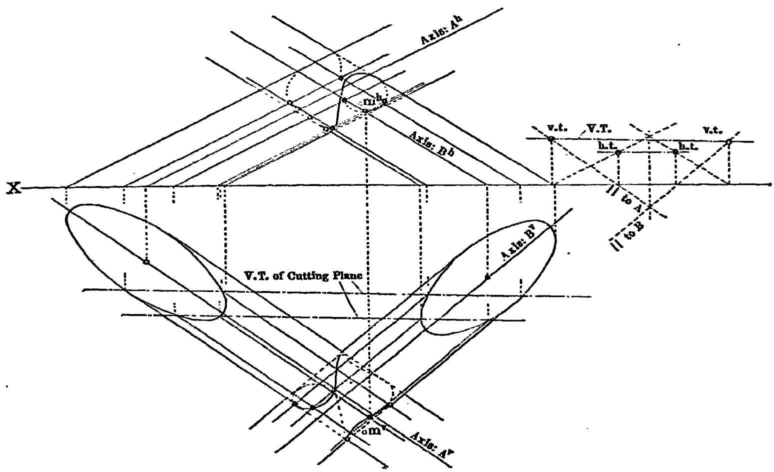


FIG. 131

in Fig. 130. This drawing is made especially pictorial in order that it may be read and studied with greater ease.

Fig. 131 shows the case of two intersecting cylinders, their axes "passing" at an angle. The cutting planes are parallel to both axes.

CHAPTER XXIV

EXAMPLE PROBLEMS

As stated in the preface, this book has been written with the one purpose of helping the student to understand the essentials of the subject. As he proceeds he should draw at least one modification of each problem solved; the simpler solutions can be drawn freehand; in fact, the freehand habit is an excellent one all through the subject; it is best to take several cases and to assume odd and peculiar conditions, in order to realize that the analyses are true for all cases. The student should also work out all the example problems given in this chapter; and, in addition, he should devise for himself simple practical problems based on the essentials as he passes them.

Teachers using this as a text book will of course give special problems suited to the general branch of engineering which the student is following.

THE THIRD ANGLE: In the following example problems, unless otherwise stated, given magnitudes are in the 3rd dihedral angle.

WITH CHAPTER III

A–1: Show, letter, and dimension the following points; their projection lines equally spaced, say 1″ apart.

Point *a*	$1\frac{3}{4}''$ below H	$1''$ behind V
" *b*	$1\frac{1}{4}''$ above H	$2''$ behind V
" *c*	$1\frac{1}{2}''$ above H	$2\frac{1}{2}''$ before V
" *d*	$1''$ below H	$1''$ before V
" *e*	$1\frac{1}{4}''$ below H	in V
" *f*	$1\frac{3}{4}''$ above H	in V
" *g*	in H	$2''$ behind V
" *j*	in H	$1''$ before V
" *k*	in H	in V

A–2: Show four points, *a*, *b*, *c*, *d*, having the same projection line and respectively in the 1st, 2nd, 3rd and 4th dihedral angles.

A–3: A point on the board 1″ above X is the H.P. of three points, *a*, *b*, *c*; *a* is in the 2nd angle; *b* is in H; *c* is in the 3rd angle.

WITH CHAPTER IV

B–1: Show the H and V projections of the following lines:
A, parallel to both H and V.
B, parallel to H, perpendicular to V.
C, parallel to H, inclined to V.
D, perpendicular to H, parallel to V.
E, inclined to H, parallel to V.
F, inclined to both H and V.
G, intersecting X.
M, in V.
N, in H.

B–2: The ends of line *a b* are in projection lines $1\frac{1}{2}$″ apart; *a* is $\frac{5}{8}$″ from H, $1\frac{1}{8}$″ from V; *b* is 1″ from H, 2″ from V; show the line.

B–3: Show the H and V projections of a triangle *abc*, *a* in the 1st angle, *b* in the 2nd, *c* in the 3rd.

WITH CHAPTER V

C–1: Assume an oblique line crossing the 3rd angle, and show its traces h.t., v.t.

C–2: The h.t. of a line is $\frac{3}{4}$″ above X, the v.t. is $2\frac{1}{8}$″ to the left of the h.t. and $1\frac{1}{4}$″ below X; show the line.

C–3: Show a line whose h.t. and v.t. are both below X. Which angle does it cross?

C–4: Show a line whose h.t. and v.t. are both in X.

C–5: In a given line *ab*, b^h is the h.t., $\frac{3}{4}$″ above X; a line C has the same h.t. and has its v.t. in a^vb^v, 1″ below X and $1\frac{3}{4}$″ to the left of b^v; the v.t. of *ab* is $\frac{5}{8}$″ above X. Show both lines.

WITH CHAPTER VI

D–1: Show the 3rd-angle portion of the following planes:

Plane 1, parallel to H, $\frac{3}{4}''$ from it.
" 2, parallel to V, $\frac{1}{2}''$ from it.
" 3, perpendicular to H, 30° to V.
" 4, perpendicular to V, 45° to H.
" 5, perpendicular to X.
" 6, parallel to X, inclined at 45° to H and V.
" 7, inclined to both H and V, intersecting X.

WITH CHAPTER VIII

E–1: Show the profile projections of points *a*, *b*, *c*, problem A–3.

E–2: Point *a* is in the 2nd angle in a line D which passes through X in a plane at right angles to both H and V. D makes 30° with H; *a* is 1″ from H. Line D also contains point *b* in the 4th angle, $3\frac{1}{2}''$ from *a*. Show *a* and *b* and dimension from direct measurements.

E–3: Assume an oblique plane of intersecting traces; show the P.T.

E–4: Show a plane parallel to X when $\theta = 30°$. What is ϕ?

E–5: Show a plane containing X.

E–6: Show the profile trace of plane 3, Problem D–1.

WITH CHAPTER IX

F–1: Draw the H.P. and V.P. of a 2″ cube, top parallel to H, faces inclined to V; show the projection on an auxiliary reference plane perpendicular to H, inclined to V.

F–2: Obtain the same projection as in F–1, by revolving the cube about suitable axis.

F–3: Assume a point and revolve it into H about axis R in H, showing both revolved positions; R should be inclined to X and should not contain the H.P. of the point.

F–4: Assume an oblique line and using one projection as axis revolve a point of the line into V.

F–5: Revolve any oblique line until parallel to H or V.

WITH CHAPTER XII

G–1: Draw a line 2″ long, one end in H, the other $\frac{3}{4}$″ from V; $\theta=30°$, $\phi=45°$

G–2: Fig. G–2 shows a 3″ cube *abc*; a hole is drilled through from *m* to *n*; what angle will this hole make with each of the three faces *ab, bc, ca*?

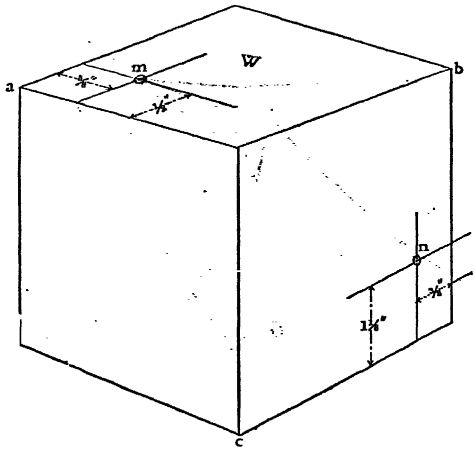


FIG. G-2

G–3: Make a completely dimensioned mechanical drawing of a triangular prism, $1\frac{1}{4}$″ sides, which will fit inside adjacent walls of a square box, and make 30° with one wall and 45° with the other; the longest edge of the prism to be 4″.

WITH CHAPTER XIII

H–1: Find θ, ϕ, and K of a plane whose H and V traces are in the same straight line making an angle of 30° with X. *Note.*—A plane whose traces are in the same straight line is illustrated in both perspective and descriptive in Fig. H–1. In handling such a plane, it must be remembered that the one line drawn is in reality *two* lines.

H–2: Find K_3 of trace plane whose $\theta=45°$, $\phi=75°$.

H–3: Make completely dimensioned mechanical drawing of a rectangular bar 2″×1″, longest edge 5″, one end cut square; the other end to make 30° with a 2″ face of bar, and to be cut so that the angle between adjacent sides of it makes 45°.

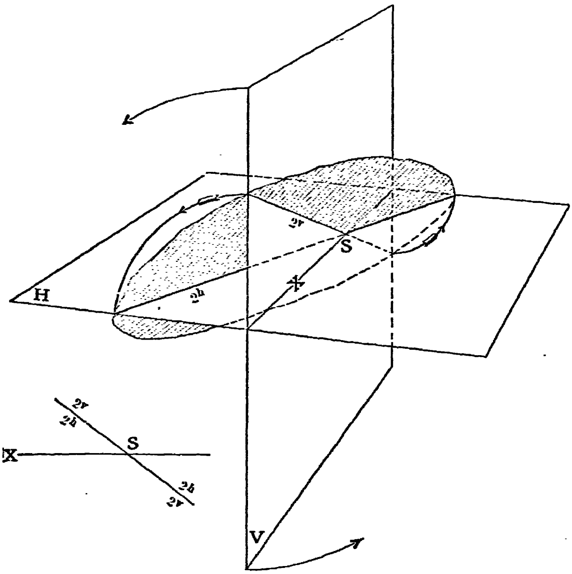


FIG. H-1

H–4: Point a square bar, size at pleasure, so that the four triangles forming the point are isosceles of 75° angles.

H–5: Cut off a corner of a cube so that the triangle obtained has sides $1\frac{1}{2}''$, $2''$ and $2\frac{1}{2}''$ long; and exhibit the angle that the surface makes with the adjacent faces of the cube.

WITH CHAPTER XIV

J–1: Assume a line; through any point in it draw another line; find the plane of the lines.

J–2: Assume a point in a given trace plane.

J–3: Pass a plane through three points *a*, *b*, *c*; *a* in the 1st angle, *b* in the 2nd, *c* in the 3rd.

J–4: Place a line on a face of an equilateral triangular prism so that it

J–5: Assume a cube, and show a trace plane at right angles to one of its diagonals.

J–6: The traces of a plane make 30° and 45° with X. Show a square pyramid with base in this plane.

WITH CHAPTER XV

K–1: Pass a plane through the center of a cube and making angles of 60° and 45° with adjacent faces. Show the true shape of this section of the cube.

K–2: Through one corner of a cube pass a plane perpendicular to the diagonal from that corner.

WITH CHAPTER XVI

L–1: Assume three trace planes, no two of which are parallel, and find their point of intersection.

L–2: Assume two oblique trace planes not parallel, and place a line 2″ long so that it just touches both planes and is parallel to X.

L–3: Draw a perspective projection of any regular solid; point of sight at pleasure.

WITH CHAPTER XVII

M–1: Exhibit the angle that a diagonal of a cube makes with an adjacent edge.

M–2: Given two oblique intersecting lines; place the line that bisects the angle α between them.

M–3: Show the H and V projections of a cube whose diagonal has $\theta=30°$, $\phi=45°$.

WITH CHAPTER XVIII

N–1: Show the projections of a square pyramid, θ of base $=60°$ and no edge horizontal.

N–2: Show the projections of a cube when θ of base $=60°$, and θ of one edge of base $=45°$.

N–3: Show projections of a cube when two adjacent edges have $\theta=30°$ and 45° respectively.

N–4: Show projections of a tetrahedron when θ of adjacent faces $=45°$ and 75° respectively.

WITH CHAPTER XIX

O–1: Show projections of three unequal spheres, each tangent to the other two and all touching V.

O–2: Show a cylinder, axis $\theta=45°$ and $\phi=30°$, enveloping a given sphere.

O–3: Show a cone, axis $\theta=45°$ and $\phi=30°$; enveloping a given sphere.

WITH CHAPTERS XX AND XXI

P–1: Show a right cone tangent to a given trace plane and having a given oblique line as axis.

P–2: Given a right cone, axis θ and ϕ; from an outside point as vertex draw a cone tangent to the given one, and show the point of tangency.

P–3: Draw a normal to the surface of a given cone from a given outside point.

P–4: Draw a normal to the surface of a given cone and parallel to a given trace plane.

P–5: Show a right cylinder tangent to a given right cylinder and having a given oblique line as axis.

P–6: Show the H.T. of a helicoidal surface, axis vertical; draw a normal to the surface.

P–7: From a point outside a given sphere pass a plane of given θ tangent to the sphere.

P–8: Pass a plane of given θ and ϕ tangent to a given sphere.

P–9: Determine the trace plane tangent to the three spheres of O–1.

P–10: Show a square-thread screw and determine the largest portion of a half-nut that can be removed without unscrewing.

WITH CHAPTER XXII

Q–1: Show the true shape of the section of a polyhedron produced by a cutting plane of given θ and ϕ.

Q–2: Show an oblique section of a helicoid.

Q–3: Show an oblique section of a hyperboloid.

Note.—As shown in Fig. 112, the hyperboloid is the surface produced by revolving a straight line about an axis not in the same plane.

WITH CHAPTER XXIII

R–1: Arrange two intersecting square prisms, axes oblique, and obtain their intersection and developments.

R–2: Arrange two intersecting pyramids, one having four sides and the other six, axes oblique; and obtain their intersection and developments.

R–3. Arrange pyramid intersecting prism and obtain their intersection and developments.

R–4: Obtain the intersection of a sphere and a right cone, and develop the cone surface.

R–5: Arrange two unequal intersecting spheres, line of centers oblique, and obtain their intersection.

R–6: Arrange two oblique cones, axes not in the same plane and obtain their intersection and developments.

R–7: Arrange a right cylinder, axis vertical, intersecting a right cylinder, axis θ and ϕ; obtain intersection and developments.

R–8: Obtain the intersection of a right cylinder and a sphere, axis of cylinder not containing center of sphere.

R–9: Arrange two right cylinders, axes oblique and not in the same plane, and obtain their intersection and developments.

R–10: Find the profile of a circular milling cutter to groove a twist drill and give a straight lip.